In this document, let $a(n)$ be the smallest prime p such that $\sum_{q\text{ prime}\le p}\left(\frac{-n}{q}\right) > 0$ ($\{a(n)\}$ be the OEIS sequence A392284). We suppose that $n = 1605^e m^2$ for odd e, $\gcd(1605, m) = 1$. We write $P = 24996190781$, and S be the set of prime factors $< P$ of m.

Claim. If $\sum_{q\in S}\left(\frac{-1605}{q}\right) \ge -9$, then other than the following cases, we have either $\min S < a(n) < \max S$ or $a(n) \ge P$:

- $S = \{2, 7\}$, $a(n) = 11$;
- $S = \{2, 13, 17, 19, 23, 29\}$, $a(n) = 31$;
- $S = \{7, 13, 17, 19, 23, 29\}$, $a(n) = 31$;
- $S = \{2, 7, 11, 13, 17, 19, 23, 29\}$, $a(n) = 31$.

Proof. The following table lists $\left(\frac{-1605}{p}\right)$ for small primes p:

p	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47
$\left(\frac{-1605}{p}\right)$	-1	0	0	-1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	-1

We note that $\sum_{q\text{ prime}\le p}\left(\frac{-1605}{q}\right)$

- ≤ -1 for $2 \le p \le P$;
- ≤ -2 for $13 \le p \le P$;
- ≤ -3 for $17 \le p \le P$;
- ≤ -4 for $19 \le p \le P$;
- ≤ -5 for $23 \le p \le P$;
- ≤ -6 for $37 \le p \le P$;
- ≤ -7 for $41 \le p \le P$;
- ≤ -8 for $43 \le p \le P$;
- ≤ -9 for $47 \le p < P$.

If the set S is empty, then

$$\sum_{q\text{ prime}\le p}\left(\frac{-n}{q}\right) = \sum_{q\text{ prime}\le p}\left(\frac{-1605}{q}\right) < 0, \quad \forall p < P,$$

and so $a(n) > P$. So let's assume that $S \ne \emptyset$. Now

$$\sum_{q\text{ prime}\le p}\left(\frac{-n}{q}\right) = \begin{cases} \sum_{q\text{ prime}\le p}\left(\frac{-1605}{q}\right) < 0, & p < \min S; \\ \sum_{q\text{ prime}\le p}\left(\frac{-1605}{q}\right) - \sum_{q\in S}\left(\frac{-1605}{q}\right), & \max S \le p < P, \end{cases}$$

We see that $a(n) > \min S$, and $\max S < a(n) < P$ is impossible in the following cases:

- $\max S \ge 47$, $\sum_{q\in S}\left(\frac{-1605}{q}\right) \ge -9$;
- $\max S \ge 43$, $\sum_{q\in S}\left(\frac{-1605}{q}\right) \ge -8$;
- $\max S \ge 41$, $\sum_{q\in S}\left(\frac{-1605}{q}\right) \ge -7$;
- $\max S \ge 37$, $\sum_{q\in S}\left(\frac{-1605}{q}\right) \ge -6$;

- $\max S \geq 23$, $\sum_{q\in S} \left(\frac{-1605}{q}\right) \geq -5$;
- $\max S \geq 19$, $\sum_{q\in S} \left(\frac{-1605}{q}\right) \geq -4$;
- $\max S \geq 17$, $\sum_{q\in S} \left(\frac{-1605}{q}\right) \geq -3$;
- $\max S \geq 13$, $\sum_{q\in S} \left(\frac{-1605}{q}\right) \geq -2$;
- $\sum_{q\in S} \left(\frac{-1605}{q}\right) \geq -1$.

Let's study the cases where none of these conditions are satisfied.

(a) $\max S < 13$, $\sum_{q\in S} \left(\frac{-1605}{q}\right) = -2$. Then we have $S = \{2, 7\}$ and $a(n) = 11$.

(b) $\max S < 17$, $\sum_{q\in S} \left(\frac{-1605}{q}\right) = -3$. Then we have $S = \{2, 7, 13\}$ and $a(n) = 11$. Here $\min S < a(n) < \max S$ is satisfied.

(c) $\max S < 19$, $\sum_{q\in S} \left(\frac{-1605}{q}\right) = -4$. Then we have $S = \{2, 7, 13, 17\}$ and $a(n) = 11$. Here $\min S < a(n) < \max S$ is satisfied.

(d) $\max S < 23$, $\sum_{q\in S} \left(\frac{-1605}{q}\right) = -5$. Then we have $S = \{2, 7, 13, 17, 19\}$ and $a(n) = 11$. Here $\min S < a(n) < \max S$ is satisfied.

(e) $\max S < 37$, $\sum_{q\in S} \left(\frac{-1605}{q}\right) = -6$. Similarly to the cases above, if $2, 7 \in S$ and $11 \notin S$, then $a(n) = 11$ and we are done. If at least one of $2 \notin S$, $7 \notin S$, and $11 \in S$ holds true, then $\sum_{q\in S} \left(\frac{-1605}{q}\right) = -6$ implies S can only be

- $S = \{2, 13, 17, 19, 23, 29\}$;
- $S = \{7, 13, 17, 19, 23, 29\}$;
- $S = \{2, 7, 11, 13, 17, 19, 23, 29\}$,

in these cases $a(n) = 31$.

(f) $\max S < 41$, $\sum_{q\in S} \left(\frac{-1605}{q}\right) = -7$. If at least one of $2 \notin S$, $7 \notin S$, and $11 \in S$ holds true, then $\sum_{q\in S} \left(\frac{-1605}{q}\right) = -7$ implies S can only be

- $S = \{2, 13, 17, 19, 23, 29, 37\}$;
- $S = \{7, 13, 17, 19, 23, 29, 37\}$;
- $S = \{2, 7, 11, 13, 17, 19, 23, 29, 37\}$,

in these cases $a(n) = 31$, and $\min S < a(n) < \max S$ is satisfied.

(g) $\max S < 43$, $\sum_{q\in S} \left(\frac{-1605}{q}\right) = -8$. If at least one of $2 \notin S$, $7 \notin S$, and $11 \in S$ holds true, then $\sum_{q\in S} \left(\frac{-1605}{q}\right) = -8$ implies S can only be

- $S = \{2, 13, 17, 19, 23, 29, 37, 41\}$;
- $S = \{7, 13, 17, 19, 23, 29, 37, 41\}$;
- $S = \{2, 7, 11, 13, 17, 19, 23, 29, 37, 41\}$,

in these cases $a(n) = 31$, and $\min S < a(n) < \max S$ is satisfied.

(h) $\max S < 47$, $\sum_{q\in S}\left(\frac{-1605}{q}\right) = -9$. If at least one of $2 \notin S$, $7 \notin S$, and $11 \in S$ holds true, then $\sum_{q\in S}\left(\frac{-1605}{q}\right) = -9$ implies S can only be

- $S = \{2, 13, 17, 19, 23, 29, 37, 41, 43\}$;
- $S = \{7, 13, 17, 19, 23, 29, 37, 41, 43\}$;
- $S = \{2, 7, 11, 13, 17, 19, 23, 29, 37, 41, 43\}$,

in these cases $a(n) = 31$, and $\min S < a(n) < \max S$ is satisfied.

$\square$