The Dihedral Group D_4 (I): Subgroups and the Cayley Table

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The sequences A381968 = r and A380817 = s generate under the operation of composition a finite non-abelian group of permutations of positive integers which is isomorphic to the dihedral group D_4. The identity element is id = A000027. The following lists the 8 elements of D_4.

- id = A000027, order 1
- r = A381968, order 4
- r^2 = A381662, order 2
- r^3 = A382499, order 4
- s = A380817, order 2
- rs = A382679, order 2
- r^2s = A376214, order 2
- r^3s = A382680, order 2

There are 10 subgroups of the group D_4.

1. **Trivial:**
   - {id} = {A000027}
   - The entire group D_4
2. **Cyclic subgroups of order 2:**
   - {id, r^2} = {A000027, A381662}
   - {id, s} = {A000027, A380817}
   - {id, rs} = {A000027, A382679}
   - {id, r^2s} = {A000027, A376214}
   - {id, r^3s} = {A000027,A382680}

3. **Cyclic Subgroup of order 4:**
   - $\{\text{id}, r, r^2, r^3\}$ = {A000027, A381968, A381662, A382499}
4. **Subgroups of order 4 (the Klein four-group):**
   - $V_4 = \{\text{id}, r^2, s, r^2 s\}$ = {A000027, A381662, A380817, A376214}
   - $V_4 = \{\text{id}, r^2, rs, r^3 s\}$ = {A000027, A381662, A382679, A382680}

Conjugacy classes list:

- $\{\text{id}\}$ = {A000027}
- $\{r^2\}$ = {A381662}
- $\{r, r^3\}$ = {A381968, A382499}
- $\{s, r^2 s\}$ = {A380817, A376214}
- $\{rs, r^3 s\}$ = {A382679, A382680}

Cayley table for the group D_4. Elements expressed in terms of generators r and s

	id	**r**	**r²**	**r³**	**s**	**rs**	**r²s**	**r³s**
id	id	r	r^2	r^3	s	rs	r^2s	r^3s
r	r	r^2	r^3	id	rs	r^2s	r^3s	s
r²	r^2	r^3	id	r	r^2s	r^3s	s	rs
r³	r^3	id	r	r^2	r^3s	s	rs	r^2s
s	s	r^3s	r^2s	rs	id	r^3	r^2	r
rs	rs	s	r^3s	r^2s	r	id	r^3	r^2
r²s	r^2s	rs	s	r^3s	r^2	r	id	r^3
r³s	r^3s	r^2s	rs	s	r^3	r^2	r	id

Cayley table for group D_4. Each element is represented by its identifier in the OEIS

	id	r	r^2	r^3	s	rs	r^2s	r^3s
	A000027	**A381968**	**A381662**	**A382499**	**A380817**	**A382679**	**A376214**	**A382680**
A000027	A000027	A381968	A381662	A382499	A380817	A382679	A376214	A382680
A381968	A381968	A381662	A382499	A000027	A382679	A376214	A382680	A380817
A381662	A381662	A382499	A000027	A381968	A376214	A382680	A380817	A382679
A382499	A382499	A000027	A381968	A381662	A382680	A380817	A382679	A376214
A380817	A380817	A382680	A376214	A382679	A000027	A382499	A381662	A381968
A382679	A382679	A380817	A382680	A376214	A381968	A000027	A382499	A381662
A376214	A376214	A382679	A380817	A382680	A381662	A381968	A000027	A382499
A382680	A382680	A376214	A382679	A380817	A382499	A381662	A381968	A000027