The Direct Product $D_4 \times C_2$: Subgroups and the Cayley Table

Boris Putievskiy

May 16 2025

The sequences A378762 (denoted h), A381968 (denoted r), and A380817 (denoted s), when interpreted as permutations of positive integers, generate under composition a finite non-abelian group. This group is isomorphic to the direct product of dihedral group D_4 and the cyclic group C_2. The identity element, denoted id, corresponds to the sequence A000027. The list of the 16 elements of that group:

- id = A000027, order 1
- r = A381968, order 4
- r^2 = A381662, order 2
- r^3 = A382499, order 4
- s = A380817, order 2
- rs = A382679, order 2
- r^2s = A376214, order 2
- r^3s = A382680, order 2
- h = A378762, order 2
- hr = A383419, order 4
- hr^2 = A383589, order 2
- hr^3 = A383590, order 4
- hs = A056023, order 2
- hrs = A383722, order 2
- hr^2s = A383723, order 2
- hr^3s = A383724, order 2

Subgroups of the group $D_4 \times C_2$

This group has 35 subgroups, listed by order.

Order 1 (1 subgroup):

1. $\{\mathrm{id}\} = \{\underline{A000027}\}$

Order 2 (11 subgroups, all C_2):

2. $\langle h \rangle = \{\mathrm{id}, h\} = \{\underline{A000027}, \underline{A378762}\}$
3. $\langle r^2 \rangle = \{\mathrm{id}, r^2\} = \{\underline{A000027}, \underline{A381662}\}$
4. $\langle s \rangle = \{\mathrm{id}, s\} = \{\underline{A000027}, \underline{A380817}\}$
5. $\langle rs \rangle = \{\mathrm{id}, rs\} = \{\underline{A000027}, \underline{A382679}\}$
6. $\langle r^2 s \rangle = \{\mathrm{id}, r^2 s\} = \{\underline{A000027}, \underline{A376214}\}$
7. $\langle r^3 s \rangle = \{\mathrm{id}, r^3 s\} = \{\underline{A000027}, \underline{A382680}\}$
8. $\langle hr^2 \rangle = \{\mathrm{id}, hr^2\} = \{\underline{A000027}, \underline{A383589}\}$
9. $\langle hs \rangle = \{\mathrm{id}, hs\} = \{\underline{A000027}, \underline{A056023}\}$
10. $\langle hrs \rangle = \{\mathrm{id}, hrs\} = \{\underline{A000027}, \underline{A383722}\}$
11. $\langle hr^2 s \rangle = \{\mathrm{id}, hr^2 s\} = \{\underline{A000027}, \underline{A383723}\}$
12. $\langle hr^3 s \rangle = \{\mathrm{id}, hr^3 s\} = \{\underline{A000027}, \underline{A383724}\}$

Order 4 (15 subgroups):

• Cyclic C_4 (2 subgroups):

13. $\langle r \rangle = \{\mathrm{id}, r, r^2, r^3\} = \{\underline{A000027}, \underline{A381968}, \underline{A381662}, \underline{A382499}\}$
14. $\langle hr \rangle = \{\mathrm{id}, hr, r^2, hr^3\} = \{\underline{A000027}, \underline{A383419}, \underline{A381662}, \underline{A383590}\}$

• Klein Four $C_2 \times C_2$ (13 subgroups):

  ◦ Subgroups of D_2 type (2):

15. $\langle r^2, s \rangle = \{\mathrm{id}, r^2, s, r^2 s\} = \{\underline{A000027}, \underline{A381662}, \underline{A380817}, \underline{A376214}\}$
16. $\langle r^2, rs \rangle = \{\mathrm{id}, r^2, rs, r^3 s\} = \{\underline{A000027}, \underline{A381662}, \underline{A382679}, \underline{A382680}\}$

  ◦ Subgroups involving h directly ($\langle h, z \rangle$ where z is order 2) (5):

17. $\langle h, r^2 \rangle = \{\mathrm{id}, h, r^2, hr^2\} = \{\underline{A000027}, \underline{A378762}, \underline{A381662}, \underline{A383589}\}$
18. $\langle h, s \rangle = \{\mathrm{id}, h, s, hs\} = \{\underline{A000027}, \underline{A378762}, \underline{A380817}, \underline{A056023}\}$

19. $\langle h, rs\rangle = \{\mathrm{id}, h, rs, hrs\} = \{\underline{A000027}, \underline{A378762}, \underline{A382679}, \underline{A383722}\}$

20. $\langle h, r^2s\rangle = \{\mathrm{id}, h, r^2s, hr^2s\} = \{\underline{A000027}, \underline{A378762}, \underline{A376214}, \underline{A383723}\}$

21. $\langle h, r^3s\rangle = \{\mathrm{id}, h, r^3s, hr^3s\} = \{\underline{A000027}, \underline{A378762}, \underline{A382680}, \underline{A383724}\}$

    ◦ Subgroups of "mixed" type (6):

22. $\langle s, hr^2\rangle = \{\mathrm{id}, s, hr^2, hr^2s\} = \{\underline{A000027}, \underline{A380817}, \underline{A383589}, \underline{A383723}\}$

23. $\langle rs, hr^2\rangle = \{\mathrm{id}, rs, hr^2, hr^3s\} = \{\underline{A000027}, \underline{A382679}, \underline{A383589}, \underline{A383724}\}$

24. $\langle r^2s, hr^2\rangle = \{\mathrm{id}, r^2s, hr^2, hs\} = \{\underline{A000027}, \underline{A376214}, \underline{A383589}, \underline{A056023}\}$

25. $\langle r^3s, hr^2\rangle = \{\mathrm{id}, r^3s, hr^2, hrs\} = \{\underline{A000027}, \underline{A382680}, \underline{A383589}, \underline{A383722}\}$

26. $\langle r^2, hs\rangle = \{\mathrm{id}, r^2, hs, hr^2s\} = \{\underline{A000027}, \underline{A381662}, \underline{A056023}, \underline{A383723}\}$

27. $\langle r^2, hrs\rangle = \{\mathrm{id}, r^2, hrs, hr^3s\} = \{\underline{A000027}, \underline{A381662}, \underline{A383722}, \underline{A383724}\}$

Order 8 (7 subgroups):

• Dihedral D_4 type (4 subgroups):

28. $\langle r, s\rangle = \{\mathrm{id}, r, r^2, r^3, s, rs, r^2s, r^3s\} = \{\underline{A000027}, \underline{A381968}, \underline{A381662}, \underline{A382499}, \underline{A380817}, \underline{A382679}, \underline{A376214}, \underline{A382680}\}$
    See The Dihedral Group D4 (I): Subgroups and the Cayley Table

29. $\langle r, hs\rangle = \{\mathrm{id}, r, r^2, r^3, hs, hrs, hr^2s, hr^3s\} = \{\underline{A000027}, \underline{A381968}, \underline{A381662}, \underline{A382499}, \underline{A056023}, \underline{A383722}, \underline{A383723}, \underline{A383724}\}$

30. $\langle hr, s\rangle = \{\mathrm{id}, hr, r^2, hr^3, s, hrs, r^2s, hr^3s\} = \{\underline{A000027}, \underline{A383419}, \underline{A381662}, \underline{A383590}, \underline{A380817}, \underline{A383722}, \underline{A376214}, \underline{A383724}\}$

31. $\langle hr, rs\rangle = \{\mathrm{id}, hr, r^2, hr^3, rs, hr^2s, r^3s, hs\} = \{\underline{A000027}, \underline{A383419}, \underline{A381662}, \underline{A383590}, \underline{A382679}, \underline{A383723}, \underline{A382680}, \underline{A056023}\}$

• Abelian $C_4 \times C_2$ type (1 subgroup):

32. $\langle h, r\rangle = \{\mathrm{id}, r, r^2, r^3, h, hr, hr^2, hr^3\} = \{\underline{A000027}, \underline{A381968}, \underline{A381662}, \underline{A382499}, \underline{A378762}, \underline{A383419}, \underline{A383589}, \underline{A383590}\}$

• Abelian $C_2 \times C_2 \times C_2$ type (2 subgroups):

33. $\langle h, r^2, s\rangle = \{\mathrm{id}, r^2, s, r^2s, h, hr^2, hs, hr^2s\} = \{\underline{A000027}, \underline{A381662}, \underline{A380817}, \underline{A376214}, \underline{A378762}, \underline{A383589}, \underline{A056023}, \underline{A383723}\}$

34. $\langle h, r^2, rs\rangle = \{\mathrm{id}, r^2, rs, r^3s, h, hr^2, hrs, hr^3s\} = \{\underline{A000027}, \underline{A381662}, \underline{A382679}, \underline{A382680}, \underline{A378762}, \underline{A383589}, \underline{A383722}, \underline{A383724}\}$

Order 16 (1 subgroup — the entire group):

35. $G = \langle h, r, s\rangle = \{\mathrm{id}, r, r^2, r^3, s, rs, r^2s, r^3s, h, hr, hr^2, hr^3, hs, hrs, hr^2s, hr^3s\} = \{\underline{A000027}, \underline{A381968}, \underline{A381662}, \underline{A382499}, \underline{A380817}, \underline{A382679}, \underline{A376214}, \underline{A382680}, \underline{A378762}, \underline{A383419}, \underline{A383589}, \underline{A383590}, \underline{A056023}, \underline{A383722}, \underline{A383723}, \underline{A383724}\}$

Conjugacy classes of the group $D_4 \times C_2$

This group has 10 conjugacy classes:

- $\{\mathrm{id}\} = \{\underline{\text{A000027}}\}$
- $\{h\} = \{\underline{\text{A378762}}\}$
- $\{r^2\} = \{\underline{\text{A376214}}\}$
- $\{hr^2\} = \{\underline{\text{A383723}}\}$
- $\{r, r^3\} = \{\underline{\text{A381968}}, \underline{\text{A382499}}\}$
- $\{hr, hr^3\} = \{\underline{\text{A383419}}, \underline{\text{A383590}}\}$
- $\{s, r^2s\} = \{\underline{\text{A380817}}, \underline{\text{A376214}}\}$
- $\{hs, hr^2s\} = \{\underline{\text{A056023}}, \underline{\text{A383723}}\}$
- $\{rs, r^3s\} = \{\underline{\text{A382679}}, \underline{\text{A382680}}\}$
- $\{hrs, hr^3s\} = \{\underline{\text{A383722}}, \underline{\text{A383724}}\}$

Cayley table for the group $D_4 \times C_2$.
Elements expressed in terms of generators h, r and s

	id	**r**	**r²**	**r³**	**s**	**rs**	**r²s**	**r³s**	**h**	**hr**	**hr²**	**hr³**	**hs**	**hrs**	**hr²s**	**hr³s**
id	id	r	r²	r³	s	rs	r²s	r³s	h	hr	hr²	hr³	hs	hrs	hr²s	hr³s
r	r	r²	r³	id	rs	r²s	r³s	s	hr	hr²	hr³	h	hr³s	hs	hrs	hr²s
r²	r²	r³	id	r	r²s	r³s	s	rs	hr²	hr³	h	hr	hr²s	hr³s	hs	hrs
r³	r³	id	r	r²	r³s	s	rs	r²s	hr³	h	hr	hr²	hrs	hr²s	hr³s	hs
s	s	r³s	r²s	rs	id	r³	r²	r	hs	hrs	hr²s	hr³s	h	hr	hr²	hr³
rs	rs	s	r³s	r²s	r	id	r³	r²	hrs	hr²s	hr³s	hs	hr³	h	hr	hr²
r²s	r²s	rs	s	r³s	r²	r	id	r³	hr²s	hr³s	hs	hrs	hr²	hr³	h	hr
r³s	r³s	r²s	rs	s	r³	r²	r	id	hr³s	hs	hrs	hr²s	hr	hr²	hr³	h
h	h	hr	hr²	hr³	hs	hrs	hr²s	hr³s	id	r	r²	r³	s	rs	r²s	r³s
hr	hr	hr²	hr³	h	hr³s	hs	hrs	hr²s	r	r²	r³	id	rs	r²s	r³s	s
hr²	hr²	hr³	h	hr	hr²s	hr³s	hs	hrs	r²	r³	id	r	r²s	r³s	s	rs
hr³	hr³	h	hr	hr²	hrs	hr²s	hr³s	hs	r³	id	r	r²	r³s	s	rs	r²s
hs	hs	hrs	hr²s	hr³s	h	hr	hr²	hr³	s	r³s	r²s	rs	id	r³	r²	r
hrs	hrs	hr²s	hr³s	hs	hr³	h	hr	hr²	rs	s	r³s	r²s	r	id	r³	r²
hr²s	hr²s	hr³s	hs	hrs	hr²	hr³	h	hr	r²s	rs	s	r³s	r²	r	id	r³
hr³s	hr³s	hs	hrs	hr²s	hr	hr²	hr³	h	r³s	r²s	rs	s	r³	r²	r	id

Cayley table for group $D_4 \times C_2$.
Each element is represented by its identifier in the OEIS

	id	r	r^2	r^3	s	rs	r^2s	r^3s	h	hr	hr^2	hr^3	hs	hrs	hr^2s	hr^3s
	A000027	**A381968**	**A381662**	**A382499**	**A380817**	**A382679**	**A376214**	**A382680**	**A378762**	**A383419**	**A383589**	**A383590**	**A056023**	**A383722**	**A383723**	**A383724**
A000027	A000027	A381968	A381662	A382499	A380817	A382679	A376214	A382680	A378762	A383419	A383589	A383590	A056023	A383722	A383723	A383724
A381968	A381968	A381662	A382499	A000027	A382679	A376214	A382680	A380817	A383419	A383589	A383590	A378762	A383724	A056023	A383722	A383723
A381662	A381662	A382499	A000027	A381968	A376214	A382680	A380817	A382679	A383589	A383590	A378762	A383419	A383723	A383724	A056023	A383722
A382499	A382499	A000027	A381968	A381662	A382680	A380817	A382679	A376214	A383590	A378762	A383419	A383589	A383722	A383723	A383724	A056023
A380817	A380817	A382680	A376214	A382679	A000027	A382499	A381662	A381968	A056023	A383722	A383723	A383724	A378762	A383419	A383589	A383590
A382679	A382679	A380817	A382680	A376214	A381968	A000027	A382499	A381662	A383722	A383723	A383724	A056023	A383590	A378762	A383419	A383589
A376214	A376214	A382679	A380817	A382680	A381662	A381968	A000027	A382499	A383723	A383724	A056023	A383722	A383589	A383590	A378762	A383419
A382680	A382680	A376214	A382679	A380817	A382499	A381662	A381968	A000027	A383724	A056023	A383722	A383723	A383419	A383589	A383590	A378762
A378762	A378762	A383419	A383589	A383590	A056023	A383722	A383723	A383724	A000027	A381968	A381662	A382499	A380817	A382679	A376214	A382680
A383419	A383419	A383589	A383590	A378762	A383724	A056023	A383722	A383723	A381968	A381662	A382499	A000027	A382679	A376214	A382680	A380817
A383589	A383589	A383590	A378762	A383419	A383723	A383724	A056023	A383722	A381662	A382499	A000027	A381968	A376214	A382680	A380817	A382679
A383590	A383590	A378762	A383419	A383589	A383722	A383723	A383724	A056023	A382499	A000027	A381968	A381662	A382680	A380817	A382679	A376214
A056023	A056023	A383722	A383723	A383724	A378762	A383419	A383589	A383590	A380817	A382680	A376214	A382679	A000027	A382499	A381662	A381968
A383722	A383722	A383723	A383724	A056023	A383590	A378762	A383419	A383589	A382679	A380817	A382680	A376214	A381968	A000027	A382499	A381662
A383723	A383723	A383724	A056023	A383722	A383589	A383590	A378762	A383419	A376214	A382679	A380817	A382680	A381662	A381968	A000027	A382499
A383724	A383724	A056023	A383722	A383723	A383419	A383589	A383590	A378762	A382680	A376214	A382679	A380817	A382499	A381662	A381968	A000027