The Lucas-Fibonacci Polynomials

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A374439

```
M := 2: # Family index

T := proc(n, k) option remember;
     if k > n then 0 elif k < M then k + 1
     else T(n - 1, k) + T(n - M, k - M) fi end:

seq(print(seq(T(n, k), k = 0..n)), n = 0..9);
```

$$
\begin{aligned}
&1\\
&1, 2\\
&1, 2, 1\\
&1, 2, 2, 2\\
&1, 2, 3, 4, 1\\
&1, 2, 4, 6, 3, 2\\
&1, 2, 5, 8, 6, 6, 1\\
&1, 2, 6, 10, 10, 12, 4, 2\\
&1, 2, 7, 12, 15, 20, 10, 8, 1\\
&1, 2, 8, 14, 21, 30, 20, 20, 5, 2
\end{aligned}
\tag{1}
$$

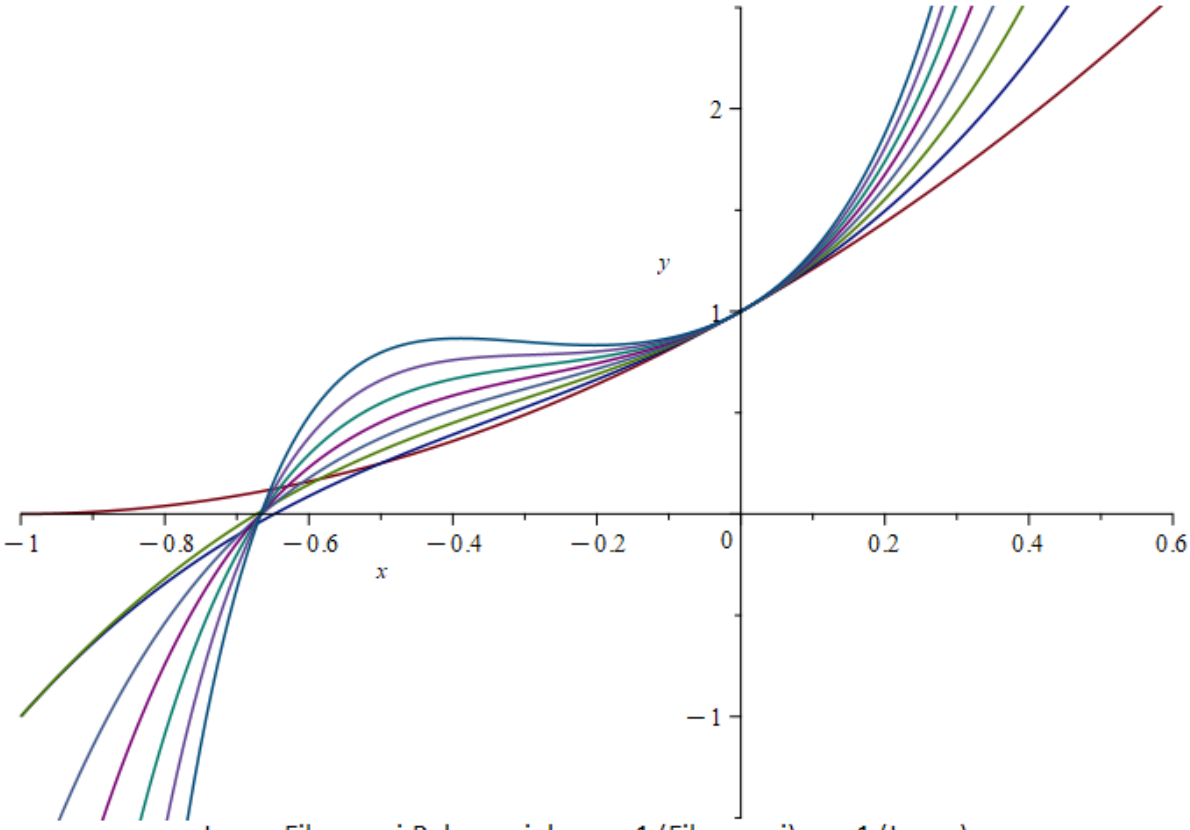
```
P := n -> local k; add(T(n, k)*x^k, k = 0..n):

seq(subs(x = 1, P(n - 1)), n = 0..19);
seq(subs(x =-1, -P(n + 1)), n = 0..19);


[Lucas A000204]
0,1,3,4,7,11,18,29,47,76,123,199,322,521,843,1364,2207,3571,...

[Fibonacci A212804, A000045]
1,0,1,1,2,3,5,8,13,21,34,55,89,144,233,377,610,987,1597,2584,...


plot([seq(P(n), n=2..9)], x = -1..0.6, y = -1.5..2.5);
```



Lucas-Fibonacci Polynomials, x = -1 (Fibonacci), x = 1 (Lucas)