Drawings for A373008

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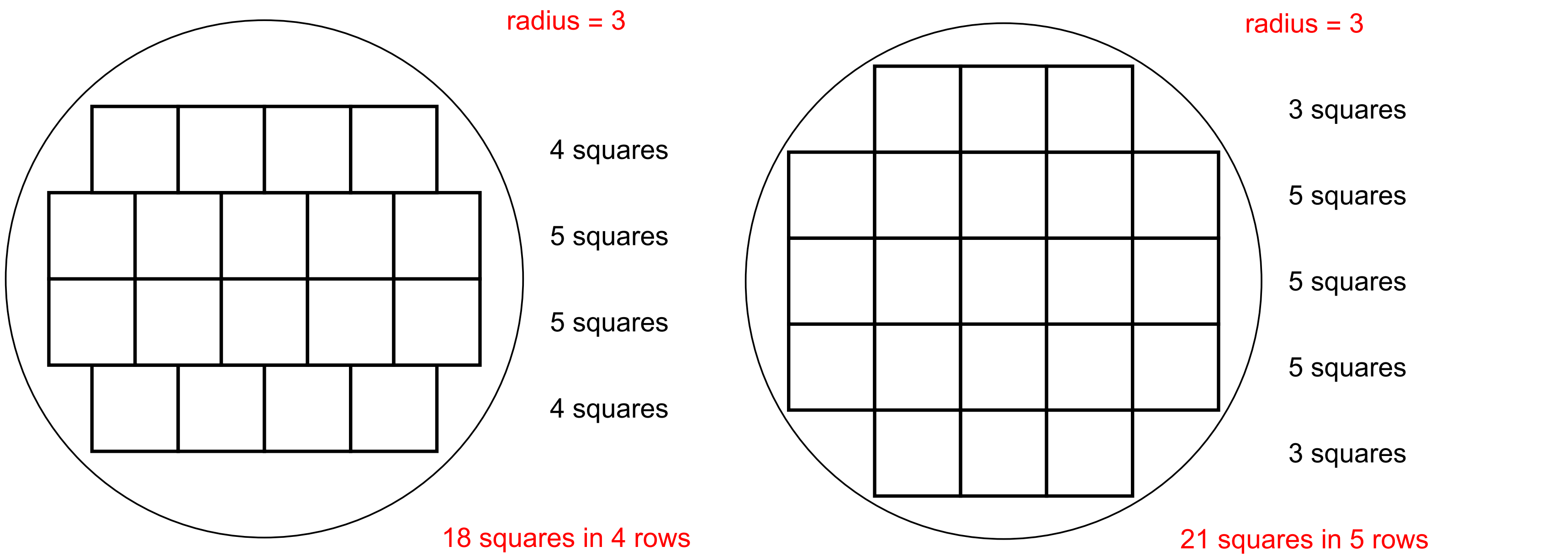
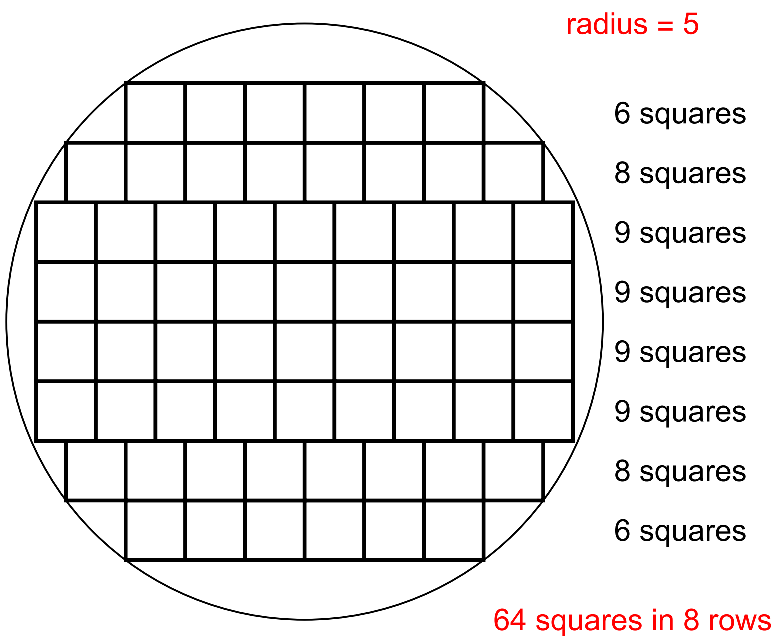
David Dewan

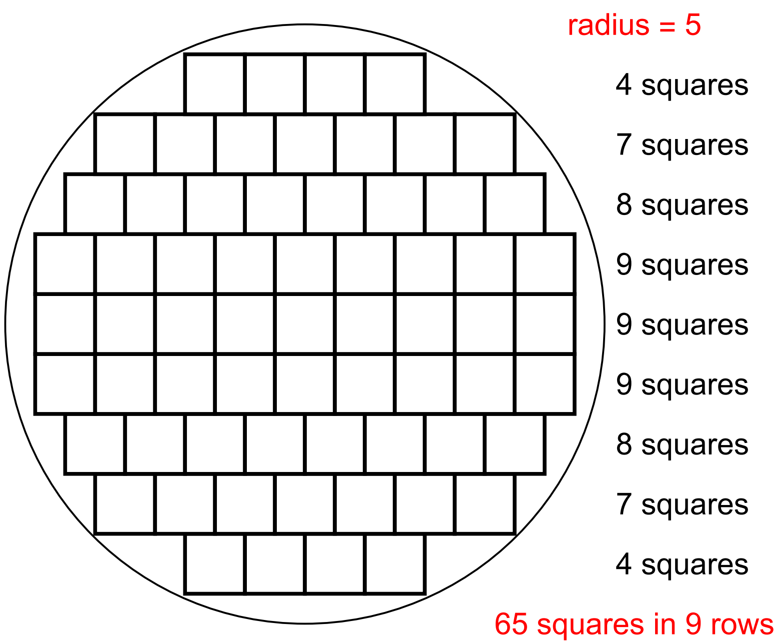
Most circles with radius r enclose fewer unit squares when they have
an even number of rows (2*r - 2) instead of a larger, odd number of rows (2*r - 1).

For example, a circle with radius = 3 only encloses 18 squares when it has 4 rows, but encloses 21 squares when it has 5 rows.

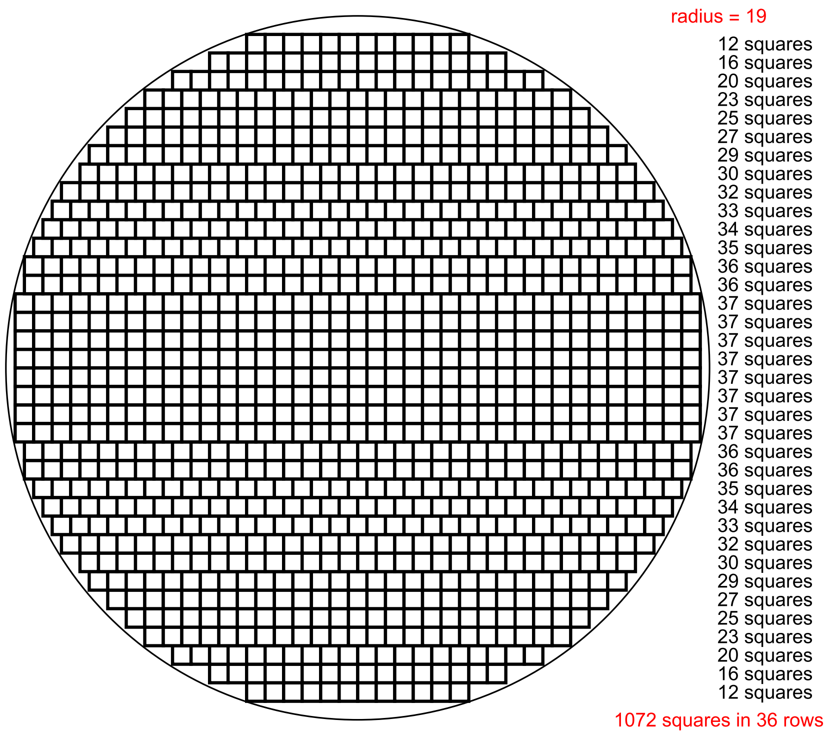


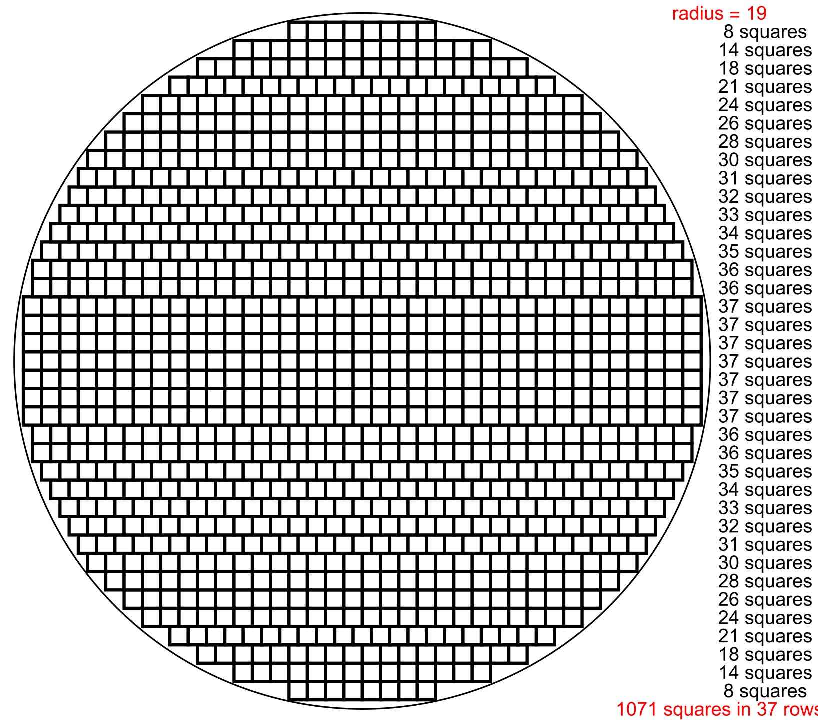
Similarly, a circle with radius = 5 only encloses 64 squares when it has 8 rows, but encloses 65 squares when it has 9 rows.





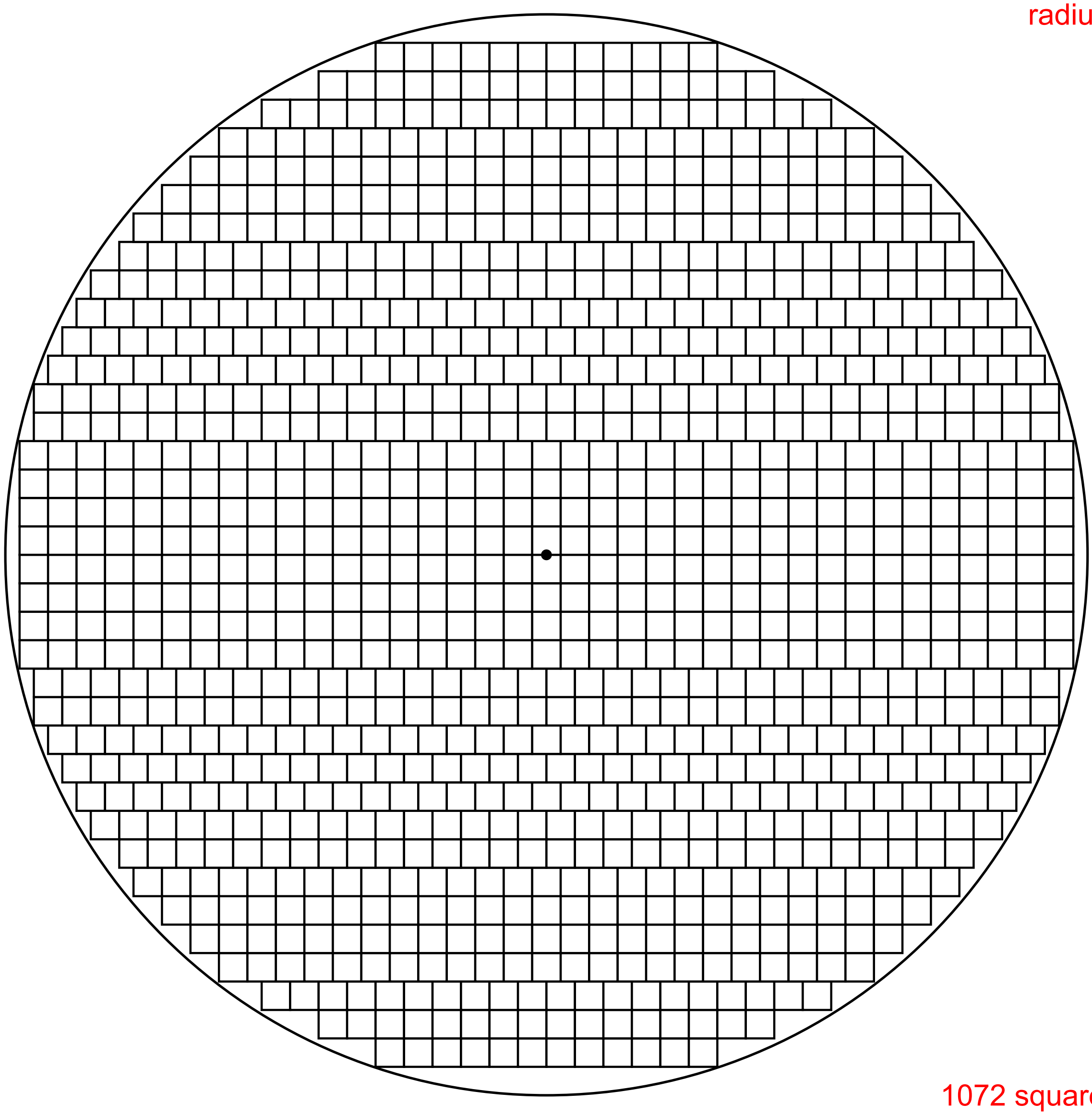
But some circles (radius = 19, for example, and others in A373008) enclose more squares when they have a **smaller** number of rows.





It looks unlikely because the 36-row version has those big gaps at the top and bottom and it’s missing the entire 37-square horizontal row in the middle, but it more than makes up for it with efficient packing elsewhere.

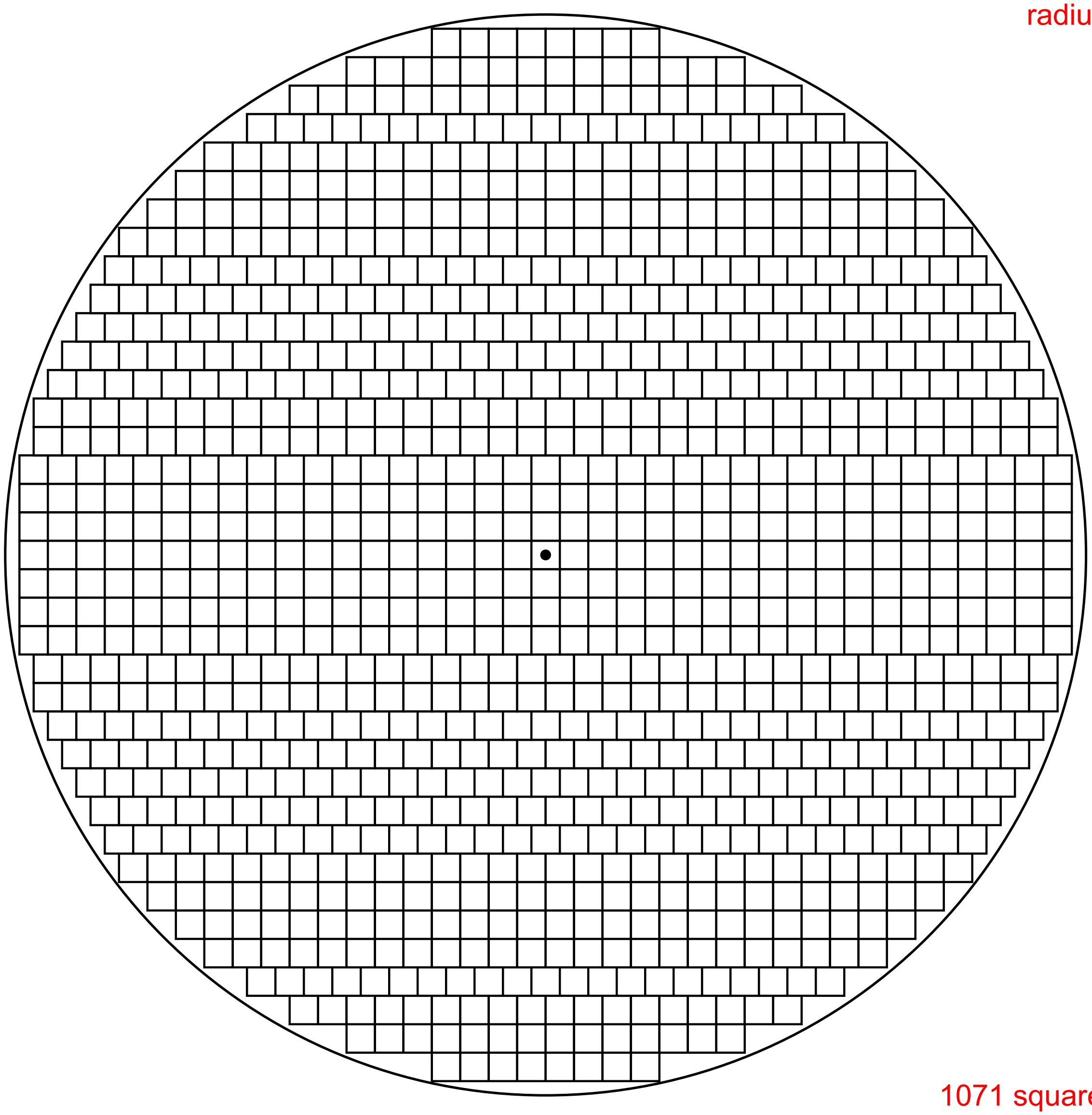
Here are larger versions for radius = 19 for additional clarity:



radius = 19

12 squares
16 squares
20 squares
23 squares
25 squares
27 squares
29 squares
30 squares
32 squares
33 squares
34 squares
35 squares
36 squares
36 squares
37 squares
37 squares
37 squares
37 squares
37 squares
37 squares
37 squares
37 squares
36 squares
36 squares
35 squares
34 squares
33 squares
32 squares
30 squares
29 squares
27 squares
25 squares
23 squares
20 squares
16 squares
12 squares

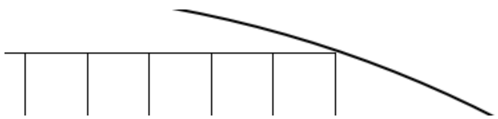
1072 squares in 36 rows



radius = 19
8 squares
14 squares
18 squares
21 squares
24 squares
26 squares
28 squares
30 squares
31 squares
32 squares
33 squares
34 squares
35 squares
36 squares
36 squares
37 squares
37 squares
37 squares
37 squares
37 squares
37 squares
37 squares
36 squares
36 squares
35 squares
34 squares
33 squares
32 squares
31 squares
30 squares
28 squares
26 squares
24 squares
21 squares
18 squares
14 squares
8 squares
1071 squares in 37 rows

Here's a closer look at a some tight fits in the 36 row diagram.

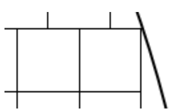
It looks close in the top row:



12 squares

But the radius is 19 and the close fit is 18 rows up: Sqrt(19^2 - 18^2) = 6.082 so there's room for 12 squares across.

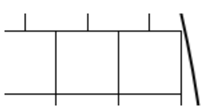
Similarly, it looks tight in the 6th row:



36 squares

The radius is 19 and it's 6 rows up: Sqrt(19^2 - 6^2) = 18.027 so there's room for 36 squares across.

It's also tight in the 4th row:



37 squares

The radius is 19 and it's 4 rows up: Sqrt(19^2 - 4^2) = 18.574 so there's room for 37 squares across.

This plot shows how many more squares are enclosed by circles having a smaller, even number of rows, for radius = 1..500.
Fewer rows are more efficient for radius = 19, 52, 65, 184, 197, 222, 230, 303, 328, 341, 425, and 489.