Sequences relating $\Omega(n)$, $\omega(n)$, and $\text{RAD}(n)$.

Sequences A205959, A363923, and A363919 of Peter Luschny.

Michael Thomas De Vlieger · St. Louis, Missouri · 11 July 2023

Abstract.

We examine related sequences that raise n to a power and dividing by squarefree kernel, where the power is the either number of distinct primes p dividing n, or those $p \mid n$ with multiplicity. These sequences exhibit behavior that excludes numbers that belong to certain easily defined subsets of the natural numbers.

Sequence A363923.

Let $\omega(n) = \text{A1221}(n)$ be the number of distinct prime factors of n, $\Omega(n) = \text{A1222}(n)$ the number of prime factors of n with multiplicity, and $\text{RAD}(n) = \text{A7947}(n)$ the squarefree kernel of n, the product of the distinct prime factors of n.

Consider Peter Luschny's sequence A363923 defined by the following equation:

$$a(n) = \text{A363923}(n) = n^{\Omega(n)}/\text{RAD}(n). \qquad [1.1]$$

The first terms of this sequence are listed below:

```
1, 1, 1, 8, 1, 6, 1, 256, 27, 10, 1, 288, 1, 14, 15,
32768, 1, 972, 1, 800, 21, 22, 1, 55296, 125, 26, 6561,
1568, 1, 900, 1, 16777216, 33, 34, 35, 279936, 1, 38,
39, 256000, 1, 1764, 1, 3872, 6075, 46, 1, 42467328,
343, 12500, 51, 5408, 1, 1417176, 55, 702464, ...
```

Peter Luschny pointed out in the comment section of A363923 that $n = 1$ or prime n implies $a(n) = 1$.

Examining the prime decomposition of these numbers with aid of Table A in the appendix and Figure 1 leads us to note the following:

$a(p) = p^1/p = 1$.

$a(p^\varepsilon) = p^{2\varepsilon}/p = p^{(2\varepsilon-1)}$.

$a(pq) = (pq)^2/(pq) = pq$, primes $p < q$.

$a(v) = v^{\Omega(v)}/v = v^{(\Omega(v)-1)}$, $v \in \text{A120944}$.

$a(t) = t^{\Omega(t)}/\text{RAD}(t)$ where $\Omega(n) \geq 3$ and $t \in \text{A126706}$.

Hence we may say generally:

For prime powers: $a(p^\varepsilon) = p^{(2\varepsilon-1)}$

and for squarefree n: $a(n) = n^{(\Omega(n)-1)}$,

$a(n)$ is never prime.

For numbers n neither squarefree nor prime powers, on account of $\Omega(n) \geq 3$, $a(n) \in \text{A246708}$.

We venture three related theorems.

Theorem A1. There are no primes in A363923.

Theorem A2. The only squarefree numbers $k > 1$ in A363923 are squarefree semiprimes.

Theorem A3. $n \in \text{A024619}$ with $\Omega(n) > 2$ implies $a(n) \in \text{A286708}$.

To prove each theorem, we approach in cases based on number of distinct prime factors and multiplicities of prime power factors. Hence we divide natural numbers $\mathbb{N}$ into $\{1\}$, the primes A40, composite prime powers A246547, squarefree composites A120994, and numbers neither squarefree nor prime powers, A126706.

Define function $M(n)$ to be the maximum exponent among prime power factors $p^\varepsilon \mid n$. Consider the following table. It is evident that aside from the empty product, the categories are mutually exclusive and break the natural numbers $\{\mathbb{N} \setminus \{1\}\}$ into 4 infinite subsets shown in the following table:

	$M(n) = 1$	$M(n) > 1$
$\omega(n) > 1$	A246547	A126706
$\omega(n) = 1$	A40	A120944

Therefore we have the following:

Prime powers A246655 = A40 ∪ A246547.
Numbers not prime powers A024619 = A120944 ∪ A126706.
Squarefree numbers A5117 = A40 ∪ A120944.
Numbers not squarefree A013929 = A246547 ∪ A126706.

Lemma 1.1. $a(1) = 1^0/1 = 1$. (Case of empty product) ■

Lemma 1.2. Prime p implies $a(p) = 1$.

Proof. $a(p) = p^1/p = 1$. (Prime case) ■

Lemma 1.3. $a(p^\varepsilon)$ is composite for $\varepsilon > 1$.

Proof. $a(p^\varepsilon) = p^{2\varepsilon}/p = p^{(2\varepsilon-1)}$. (Prime power case) ■

Lemma 1.4. Squarefree semiprimes are fixed points.

Proof: $a(pq) = (pq)^2/(pq) = pq$, primes $p < q$. (Squarefree semiprime case) ■

Lemma 1.5. Squarefree n with $\omega(n) > 2$ implies $a(n)$ composite, with power factors p^ε where $\varepsilon > 1$. ($n \in \text{A350352}$ implies $a(n) \in \text{A286708}$).

Proof:

Observe A350352 = A120944 \ A6881.

Squarefree n with $\omega(n) > 2$ implies $\Omega(n) > 2$, since for squarefree numbers, $\omega(n) = \Omega(n)$.

Squarefree n implies $\text{RAD}(n) = n$.

Therefore $a(n) = n^{\Omega(n)}/n = n^{(\Omega(n)-1)}$, and since $\Omega(n) > 2$, all power factors of $a(n)$ have multiplicity exceeding 1. (Balance of squarefree cases) ■

Lemma 1.6. Numbers n neither squarefree nor prime powers have $a(n)$composite, with power factors p^ε where $\varepsilon > 1$. ($n \in \text{A126708}$ implies $a(n) \in \text{A286708}$)

Proof:

$n \in \text{A126708}$ implies both $\Omega(n) > \omega(n)$ and $n > \text{RAD}(n)$, the latter since $n = m \times \text{RAD}(n)$, where $m > 1$.

$n \in \text{A126708}$ implies $\Omega(n) \geq 3$.

Then $a(n) = n^{\Omega(n)}/\text{RAD}(n) = m \times n^{(\Omega(n)-1)}$, and it is clear that since we have $a(n) \geq mn^2$, $a(n)$ is composite with prime power factors whose exponents exceed 1. (Balance of all cases) ■

Since Lemmas 1.1 through 1.6 cover all natural numbers, we show theorems A1, A2, and A3 to be true.

Conclusions regarding $a = \text{A363923}$:

- $1 \to 1$ and $p \to 1$; generally $p^\varepsilon \to p^{(\varepsilon-1)}$.
- $pq \to pq$; fixed points are $\{1, pq\}$.
- For $n \in \text{A350352}$, $n \to a(n)$ that is composite, with power factors $p^\varepsilon \mid a(n)$ where $\varepsilon > 1$. Therefore $a(n) \in \text{A303606}$, and since $\text{A303606} \subset \text{A286708}$, $a(n) \in \text{A286708}$.
- For $n \in \text{A126706}$, $n = m \times \text{RAD}(n) \to m^k \times \text{RAD}(n)^{(k-1)}$, $k > 1$.
- $a(n) \in \text{A1694}$ for $n \notin \text{A6881}$.

Sequence A205959.

We move on to another, earlier sequence of Luschny related to a, defined below:

$$s(n) = \text{A205959}(n) = n^{\omega(n)}/\text{RAD}(n). \qquad [2.1]$$

The first terms of this sequence are listed below:

```
1, 1, 1, 2, 1, 6, 1, 4, 3, 10, 1, 24, 1, 14, 15, 8, 1,
54, 1, 40, 21, 22, 1, 96, 5, 26, 9, 56, 1, 900, 1, 16,
33, 34, 35, 216, 1, 38, 39, 160, 1, 1764, 1, 88, 135,
46, 1, 384, 7, 250, 51, 104, 1, 486, 55, 224, 57, ...
```

Looking at Figure 2 in appendix, we note the following:

- $s(1) = s(p) = 1$.
- $s(p^\varepsilon) = p^{(\varepsilon-1)}$, $s(p^2) = p$.
- $s(pq) = pq$.

Now we attempt to write theorems similar to those in the last section pertaining to A363923, but this time for A205959. We begin by-dividing $\{\mathbb{N} \setminus \{1\}\}$ into the same infinite subsets as we had regarding A363923.

Lemma 2.1: Prime p implies $s(p) = 1$.
Proof: Suppose the proposition is true. We have the following:

$$\begin{aligned} s(p) &= p^{\omega(p)}/\text{RAD}(p) \\ &= p^1/p \\ &= 1. \end{aligned}$$

This confirms the proposition. ■ (Prime case.)

Lemma 2.2: Prime power p^ε implies $s(p^\varepsilon) = p^{(\varepsilon-1)}$.
Proof: Suppose the proposition is true. We have the following:

$$\begin{aligned} s(p^\varepsilon) &= (p^\varepsilon)^{\omega(p)}/\text{RAD}(p^\varepsilon) \\ &= (p^\varepsilon)^1/p \\ &= p^{(\varepsilon-1)}, \end{aligned}$$

confirming the proposition. ■ (Prime power case.)

Remark 2.3: We observe the special case that generates prime p:

$$s(p^2) = p.$$

Lemma 2.4: Squarefree semiprimes pq, primes $p < q$, represent fixed points in s.
Proof:

$$\begin{aligned} s(pq) &= (pq)^{\omega(pq)}/\text{RAD}(pq) \\ &= (pq)^2/(pq) \\ &= pq. \end{aligned}$$

■ (Squarefree semiprime case.)

Lemma 2.5: For $n \in$ A350352, $s(n) = n^{(\omega(n)-1)}$.
Proof: Observe that A350352 = A120944 \ A6881.
Numbers $n \in$ A350352 are squarefree with $\omega(n) \ge 3$.
Squarefree n implies $n = \text{RAD}(n)$. Therefore:

$$\begin{aligned} s(n) &= n^{\omega(n)}/\text{RAD}(n) \\ &= n^k/n \text{ where } k = \omega(n) \text{ and } k > 2 \\ &= n^{(k-1)}. \end{aligned}$$

■ (Balance of squarefree case.)

We remark that $s(n) \ge n^2$.

Lemma 2.6: For $n \in$ A126706, $s(n) > n$.
Proof: For $n \in$ A126706, both $\omega(n) = k > 1$ and $n > \text{RAD}(n)$. More precisely, $n = m \times \text{RAD}(n)$ with $m > 1$.

$$\begin{aligned} s(n) &= n^{\omega(n)}/\text{RAD}(n) \\ &= (m \times \text{RAD}(n))^k/\text{RAD}(n) \\ &= m^k \times \text{RAD}(n)^k/\text{RAD}(n) \\ &= m^k \times \text{RAD}(n)^{(k-1)} \\ &= m \times n^{(k-1)}. \end{aligned}$$

It is clear that $m \times n^{(k-1)} > n$. ■ (Final case.)

Example: $12 = 2 \times \text{RAD}(12) = 2 \times 6$, hence

$$\begin{aligned} s(12) &= 12^2/6 \\ &= 2^2 \times 6 \\ &= 24. \end{aligned}$$

Conclusions regarding s = A205959:

- $1 \to 1$ and $p \to 1$.
- $p^2 \to p$ and generally $p^\varepsilon \to p^{(\varepsilon-1)}$.
- $pq \to pq$.
- $n \to n^{(k-1)}$ for $n \in$ A350352. Thus $s(n) \in$ {A303606 \ A085986}, and since A303606 ⊂ A286708, $s(n) \in$ A286708.
- $n \to m \times n^{(k-1)}$, $m > 1$, $k > 1$, for $n \in$ A126706.

Sequence A363919.

Now finally we turn to a question of Peter Luschny. Consider the following ratio:

$$\begin{aligned} t(n) &= \text{A363919}(n) \\ &= \text{A363923}(n)/\text{A205959}(n) \\ &= (n^{\Omega(n)}/\text{RAD}(n)) \;/\; (n^{\omega(n)}/\text{RAD}(n)) \\ &= n^{\Omega(n)}/n^{\omega(n)} \\ &= n^{(\Omega(n)-\omega(n))}. \end{aligned}$$

The first terms of this sequence are listed below:

```
1, 1, 1, 4, 1, 1, 1, 64, 9, 1, 1, 12, 1, 1, 1, 4096,
1, 18, 1, 20, 1, 1, 1, 576, 25, 1, 729, 28, 1, 1, 1,
1048576, 1, 1, 1, 1296, 1, 1, 1, 1600, 1, 1, 1, 44, 45,
1, 1, 110592, 49, 50, 1, 52, 1, 2916, 1, 3136, 1, ...
```

These terms are plotted in Figure 3 in the appendix.

A question of Luschny regarded identification of indices R such that $t(R)$ sets a record. Our approach involves identifying a class of number N such that $t(N)/N > t(n)/n$ for n in any other class. We employ the infinite subsets we considered in the previous sections, but we conflate the squarefree cases as they are simplified in A363919.

Lemma 3.1: Squarefree $n \in$ A5117 implies $t(n) = 1$.
Proof: Squarefree $n \in$ A5117 implies $\omega(n) = \Omega(n)$, leaving us with the following which confirms the proposition:

$$\begin{aligned} t(n) &= n^{(\Omega(n)-\omega(n))} \\ &= n^0 \\ &= 1. \end{aligned}$$

■

Corollary 3.2: $t(1) = t(p) = 1$.

Lemma 3.3: Prime power $n = p^\varepsilon$ implies $t(p^\varepsilon) = p^{\text{A2378}(\varepsilon-1)}$.
Proof:

$$\begin{aligned} t(p^\varepsilon) &= (p^\varepsilon)^{(\Omega(p^\varepsilon)-\omega(p^\varepsilon))} \\ &= (p^\varepsilon)^{(\varepsilon-1)} \\ &= p^{(\varepsilon \times (\varepsilon-1))} \\ &= p^{\text{A2378}(\varepsilon-1)}. \end{aligned}$$

■

We observe that $(\varepsilon \times (\varepsilon-1)) > \varepsilon$ for $\varepsilon > 1$, hence $t(p^\varepsilon) \ge \Omega(p^\varepsilon)$.

Corollary 3.4: $t(p^2) = p^2$.

Lemma 3.5: A number $n \in$ A126706 that are neither squarefree nor prime power implies $t(n) > n$.
Proof: The number $n \in$ A126706 implies $\Omega(n) > \omega(n) > 1$. Let $k = \Omega(n) - \omega(n)$. It is clear that $\Omega(n) > k > 1$. Rewriting the following:

$$\begin{aligned} t(n) &= n^{(\Omega(n)-\omega(n))} \\ &= n^k \end{aligned}$$

since $k > 1$, it is clear that $n^k > n$. ■

Corollary 3.6: We have fixed points for $n \in$ A126706 such that, for some prime $p \mid n$, $n = p$ rad(n), since $\Omega(n) - \omega(n) = 1$. These are n in A072357. Therefore, 12, 18, 20, etc. represent fixed points in t.

Conclusions involving $t =$ A363919(n):

- $n \to 1$ for $n \in$ A5117.
- $p^2 \to p^2$ and generally $p^\varepsilon \to p^{\text{A2378}(\varepsilon-1)}$.
- $pq \to pq$.
- $n \to n^k$, $\Omega(n) > k > 1$ for $n \in$ A126706.
- $n \to n$ for $n \in$ A072357.

Numbers that Set Records in A363919.

The sequence R of recordsetters begins as follows:

```
1, 4, 8, 16, 32, 64, 128, 512, 1024, 2048, 4096, ...
```

The number 1 sets a record since 1 is squarefree and $1 \to 1$.

The number 4 sets the next record because 2 and 3 are also squarefree and yield 1, but $4 \to 4$, as $4 = 2^2$, the square of the smallest prime.

8 follows 4 in the sequence of records because 5, 6, and 7 are squarefree, and $8 \to 2^{(2 \times 3)} = 64$.

Theorem 4. Numbers R $\in$ A151821 imply t(R) is a local maximum.
Proof: The first record $R_1 = 1$ pertains to the case of $n \to 1$ for squarefree $n \in$ A5117 by Lemma 3.1. Thereafter, we are concerned with prime powers (Lemma 3.3) and numbers neither squarefree nor prime power (Lemma 3.5). Attempting to minimize the magnitude of these species, we are concerned with $2^{(\varepsilon+1)}$ and $(2^\varepsilon \times 3)$.

As to Lemma 3.3, since it is clear that $p^\varepsilon > 2^{(\varepsilon+1)}$ for $\varepsilon > 1$ and $p > 2$, we need not consider powers other than those of 2.

Regarding Lemma 3.5, we want multiplicity to apply to the smallest prime factor of $n \in$ A126706, and that we want to ensure the smallest possible product $n \in$ A126706 by the fewest number of distinct primes, where the primes are the very smallest such. Hence we are talking about $(2^\varepsilon \times 3)$.

Observe that $2^{(\varepsilon+1)} < (2^\varepsilon \times 3) < 2^{(\varepsilon+2)}$. The power $2^{(\varepsilon+1)}$ conforms to Lemma 4.3, hence the following transformation:

$$2^{(\varepsilon+1)} \to 2^{\varepsilon(\varepsilon+1)}.$$

The number $(2^\varepsilon \times 3)$ conforms to Lemma 3.5, and we have the following transformation:

$$(2^\varepsilon \times 3) \to 2^{\varepsilon(\varepsilon-1)} \times 3^{(\varepsilon-1)} \text{ via the following:}$$

$$\begin{aligned} t(n) &= n^{(\Omega(n) - \omega(n))} \\ &= (2^\varepsilon \times 3)^{(\varepsilon+1-2)} \\ &= 2^{\varepsilon(\varepsilon-1)} \times 3^{(\varepsilon-1)} \end{aligned}$$

For $\varepsilon \geq 1$ the following is true:

$$\begin{aligned} 2^{\varepsilon(\varepsilon+1)} &> 2^{\varepsilon(\varepsilon-1)} \times 3^{(\varepsilon-1)} \\ 2^{(\varepsilon^2+\varepsilon)} &> 2^{(\varepsilon^2-\varepsilon)} \times 3^{(\varepsilon-1)} \\ 2^{2\varepsilon} &> 3^{(\varepsilon-1)}. \end{aligned}$$

Therefore, given these facts and Lemmas 3.1-3.5, we show that record setters in t are tantamount to A151821. This is to say the following:

$$t(\mathrm{R}_i) \to \mathrm{A151821}(i). \blacksquare$$

Example: suppose $\varepsilon = 3$:

$2^{(\varepsilon+1)} \to 2^{(\varepsilon \times (\varepsilon+1))}$ becomes $16 \to 4096$ while
$(2^\varepsilon \times 3) \to (2^{\varepsilon(\varepsilon-1)} \times 3^{(\varepsilon-1)})$ becomes $24 \to 64 \times 9 = 576$.

The first few transformations appear in the table below:

```
ε          t(2^(ε+1))           t(2^ε × 3)
-------------------------------------------------
0              1 = 2^0
1              4 = 2^2               1
2             64 = 2^6              12 = 2^2 × 3
3           4096 = 2^12            576 = 2^6 × 3^2
4        1048576 = 2^20         110592 = 2^12 × 3^3
5     1073741824 = 2^30       84934656 = 2^20 × 3^4
6  4398046511104 = 2^42   260919263232 = 2^30 × 3^5
```

Hence we have demonstrated that the maxima of the sequence $t(n)$ are R $\in$ A151821.

Conclusion.

Elementary number-theoretical functions $\omega(n)$, $\Omega(n)$, and rad(n) are sensitive to prime decomposition of n. Sequences A205959, A363923, and A363919 explore the relationship of these functions. Therefore they are amenable to the partitioning of natural numbers into species predicated on two axes, that is, whether or not prime power, and whether or not squarefree. Hence we divide the natural numbers $n > 1$ into infinite subsets of the primes (A40), composite prime powers (A246547), squarefree composites (A120944), and numbers neither squarefree nor prime powers (A126706). With these species, we may approach theorems by case.

We summarize findings below:

Conclusions regarding $a =$ A363923:

- $1 \to 1$ and $p \to 1$; generally $p^\varepsilon \to p^{(\varepsilon-1)}$.
- $pq \to pq$; fixed points are $\{1, pq\}$.
- For $n \in$ A350352, $n \to a(n)$ that is composite, with power factors $p^\varepsilon \mid a(n)$ where $\varepsilon > 1$. Therefore $a(n) \in$ A303606, and since A303606 $\subset$ A286708, $a(n) \in$ A286708.
- For $n \in$ A126706, $n = m \times$ rad$(n) \to m^k \times$ rad$(n)^{(k-1)}$, $k > 1$.
- $a(n) \in$ A1694 for $n \notin$ A6881.

Conclusions regarding $s =$ A205959:

- $1 \to 1$ and $p \to 1$.
- $p^2 \to p$ and generally $p^\varepsilon \to p^{(\varepsilon-1)}$.
- $pq \to pq$.
- $n \to n^{(k-1)}$ for $n \in$ A350352. Thus $s(n) \in$ {A303606 \ A085986}, and since A303606 $\subset$ A286708, $s(n) \in$ A286708.
- $n \to m \times n^{(k-1)}$, $m > 1$, $k > 1$, for $n \in$ A126706.

Conclusions involving $t =$ A363919(n):

- $n \to 1$ for $n \in$ A5117.
- $p^2 \to p^2$ and generally $p^\varepsilon \to p^{\text{A2378}(\varepsilon-1)}$.
- $pq \to pq$.
- $n \to n^k$, $\Omega(n) > k > 1$ for $n \in$ A126706.
- $n \to n$ for $n \in$ A072357.

Records in A363919 are in A151821.

Appendix:

Table A.

n	a(n)	A067255(a(n)) delimited by "."
1	1	0
2	1	0
3	1	0
4	8	3
5	1	0
6	6	1.1
7	1	0
8	256	8
9	27	0.3
10	10	1.0.1
11	1	0
12	288	5.2
13	1	0
14	14	1.0.0.1
15	15	0.1.1
16	32768	15
17	1	0
18	972	2.5
19	1	0
20	800	5.0.2
21	21	0.1.0.1
22	22	1.0.0.0.1
23	1	0
24	55296	11.3
25	125	0.0.3
26	26	1.0.0.0.0.1
27	6561	0.8
28	1568	5.0.0.2
29	1	0
30	900	2.2.2
31	1	0
32	16777216	24
33	33	0.1.0.0.1
34	34	1.0.0.0.0.0.1
35	35	0.0.1.1
36	279936	7.7
37	1	0
38	38	1.0.0.0.0.0.0.1
39	39	0.1.0.0.0.1
40	256000	11.0.3
41	1	0
42	1764	2.2.0.2
43	1	0
44	3872	5.0.0.0.2
45	6075	0.5.2
46	46	1.0.0.0.0.0.0.0.1
47	1	0
48	42467328	19.4
49	343	0.0.0.3
50	12500	2.0.5
51	51	0.1.0.0.0.0.1
52	5408	5.0.0.0.0.2
53	1	0
54	1417176	3.11
55	55	0.0.1.0.1
56	702464	11.0.0.3
57	57	0.1.0.0.0.0.0.1
58	58	1.0.0.0.0.0.0.0.0.1
59	1	0
60	432000	7.3.3

Code:

[c1] Generate $a(n)$.

```
Array[#^PrimeOmega[#]/(Times @@
  FactorInteger[#][[All, 1]]) &, 2^10]
```

[c2] Generate $s(n)$.

```
Array[#^PrimeNu[#]/(Times @@
  FactorInteger[#][[All, 1]]) &, 2^10]
```

[c3] Generate $t(n)$.

```
Array[#^(PrimeOmega[#] - PrimeNu[#]) &, 2^10]
```

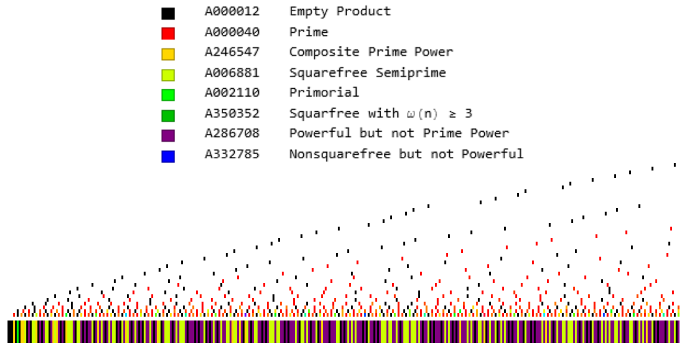

Figure 1: Plot p_k^ε | A363923(n) at $(x, y) = (n, k)$ for $n \le 360$, 2× vertical exaggeration. We use a color function in all these figures to represent ε, where black represents $\varepsilon = 1$, red $\varepsilon = 2$, ..., indigo for the maximum multiplicity ε. The bar of color at the bottom of the plot represents the class to which n belongs; the colors in this bar are according to the key above.

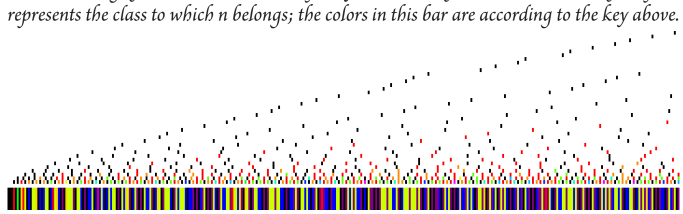

Figure 2: Plot p_k^ε | A205959(n) at $(x, y) = (n, k)$ for $n \le 360$, 2× vertical exaggeration.

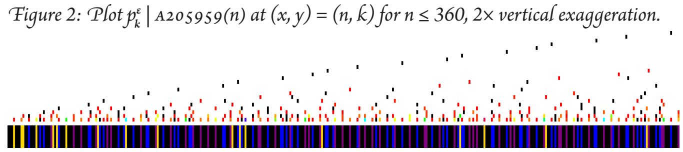

Figure 3: Plot p_k^ε | A363919(n) at $(x, y) = (n, k)$ for $n \le 360$, 2× vertical exaggeration.

References:

[1] N. J. A. Sloane, *The Online Encyclopedia of Integer Sequences*, retrieved July 2023.

Concerns sequences:

A000040: Prime numbers.
A001221: Number of distinct prime divisors of n, $\omega(n)$.
A001222: Number of prime divisors of n with multiplicity, $\Omega(n)$.
A001694: Powerful numbers, $\{a^2b^3 : a \ge 1, b \ge 1\}$.
A005117: Squarefree numbers.
A006881: Squarefree semiprimes.
A007947: Squarefree kernel of n; $\mathrm{RAD}(n)$.
A013929: Numbers that are not squarefree.
A024619: Numbers that are not prime powers.
A120944: Squarefree composites.
A126706: Numbers neither prime power nor squarefree.
A151821: $\{1\} \cup \{2^\varepsilon : \varepsilon > 1\}$.
A205959: $s(n) = n^{\omega(n)}/\mathrm{RAD}(n)$.
A246547: Composite prime powers $p^\varepsilon : \varepsilon \ge 1$.
A286708: Products of A246547.
A332785: A126706 \ A286708.
A350352: A120944 \ A006881.
A363919: $t(n) = n^{(\Omega(n) - \omega(n))}$.
A363923: $a(n) = n^{\Omega(n)}/\mathrm{RAD}(n)$.

Document Revision Record:

2023 0714: Version 1.