

Trunk Generating Functions for n -ominoes of Dimension $n-3$

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Golomb [1, pages 81-84] shows that we can represent multidimensional polyominoes as graphs, each vertex of the graph being the center of a cell, and each edge being the line segment connecting the centers of adjacent cells. The dimension of the graph (and its source polyomino) is that of the convex hull of the cell centers. Golomb notes that the number of unoriented n -ominoes of dimension $n-1$ is equivalent to the number of free trees with n nodes ([A000055](#)). For unoriented polyominoes, chiral pairs are counted as one.

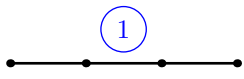
To enumerate n -ominoes of dimension $n-3$, we want to reduce the dimension of each polyomino graph by a removal process in order to obtain a small number of irreducible canonical graphs that we shall call *trunks*. First we align the graph edges with the Cartesian coordinate axes. In the removal process, an edge that is not parallel to any other edge is removed by changing the coordinate along an axis parallel to it to be the same for all vertices. In the first removal process, we remove any vertex and the single edge to which it is attached if that edge is not parallel to another edge of the graph. Our second removal involves a vertex connected to exactly two edges, neither of which is parallel to any other edge. We can eliminate this vertex by removing just one of the two edges. We shall not do this if it causes a new edge to arise elsewhere by bringing two other vertices into proximity. Note that each removal reduces the order of the polyomino by one and also reduces its dimension by one, so that the difference between the order of the polyomino and its dimension is unchanged.

It is not difficult to determine all the trunks that can arise for n -ominoes of dimension $n-3$. For each trunk we can determine a generating function that will indicate the exact number of n -ominoes that reduce to that trunk. There are separate generating functions for unoriented, chiral, and asymmetric n -ominoes. In addition, we can distinguish trunks of orthoplex polyominoes from those that have at least one coordinate with more than two values, which we call *extended* polyominoes. We sum together the generating functions for each trunk to obtain the generating function for all n -ominoes of dimension $n-3$. There are 56 trunks for orthoplex polyominoes and 173 for extended polyominoes. For multidimensional polyominoes we use all 229 trunks.

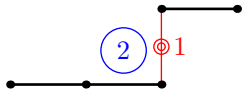
We have done this process for n -ominoes of dimension $n-2$. The results for multidimensional polyominoes are [A036364](#) (unoriented), [A036365](#) (chiral), and [A036366](#) (asymmetric). For orthoplex polyominoes we have [A036367](#) (unoriented), [A036368](#) (chiral), and [A036369](#) (asymmetric).

References

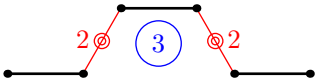
- [1] S. W. Golomb. *Polyominoes*. Princeton University Press, second edition, 1994.



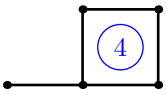
$$(A(x)^4 - A(x^2)^2) / 2 = 2x^5 + 4x^6 + 12x^7 + 26x^8 + 62x^9 + 138x^{10} + 322x^{11} + 732x^{12} + 1698x^{13} + 3918x^{14} + 9130x^{15} + 21278x^{16} + 49878x^{17} + 117126x^{18} + 276102x^{19} + 652270x^{20} + \dots$$



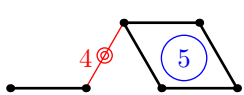
$$A(x)^5 / (1 - A(x)) = x^5 + 6x^6 + 22x^7 + 69x^8 + 199x^9 + 546x^{10} + 1457x^{11} + 3819x^{12} + 9886x^{13} + 25371x^{14} + 64711x^{15} + 164300x^{16} + 415729x^{17} + 1049176x^{18} + 2642443x^{19} + 6644669x^{20} + \dots$$



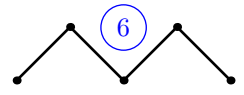
$$(A(x)^6 / (1 - A(x))^2 - A(x^2)^3 / (1 - A(x^2))) / 2 = 4x^7 + 17x^8 + 73x^9 + 244x^{10} + 792x^{11} + 2380x^{12} + 6997x^{13} + 19905x^{14} + 55770x^{15} + 153613x^{16} + 418719x^{17} + 1129797x^{18} + 3026251x^{19} + 8052108x^{20} + \dots$$



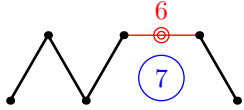
$$A(x)^5 = x^5 + 5x^6 + 15x^7 + 40x^8 + 100x^9 + 241x^{10} + 575x^{11} + 1365x^{12} + 3230x^{13} + 7640x^{14} + 18086x^{15} + 42865x^{16} + 101745x^{17} + 241895x^{18} + 576030x^{19} + 1373937x^{20} + \dots$$



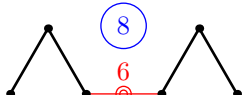
$$A(x)^6 / (1 - A(x)) = x^6 + 7x^7 + 29x^8 + 99x^9 + 305x^{10} + 882x^{11} + 2454x^{12} + 6656x^{13} + 17731x^{14} + 46625x^{15} + 121435x^{16} + 313984x^{17} + 807281x^{18} + 2066413x^{19} + 5270732x^{20} + \dots$$



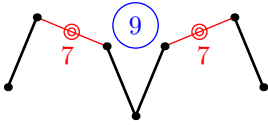
$$A(x) (A(x)^4 - A(x^2)^2) / 2 = 2x^6 + 6x^7 + 18x^8 + 46x^9 + 114x^{10} + 274x^{11} + 658x^{12} + 1564x^{13} + 3718x^{14} + 8824x^{15} + 20968x^{16} + 49848x^{17} + 118692x^{18} + 282942x^{19} + 675526x^{20} + \dots$$



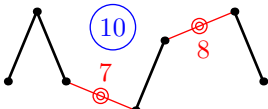
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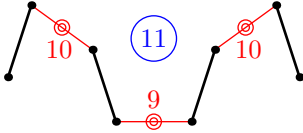
$$(A(x)^6 / (1 - A(x)) - (1 + A(x))A(x^2)^3 / (1 - A(x^2))) / 2 = 3x^7 + 12x^8 + 47x^9 + 144x^{10} + 432x^{11} + 1200x^{12} + 3296x^{13} + 8779x^{14} + 23195x^{15} + 60422x^{16} + 156528x^{17} + 402529x^{18} + 1031216x^{19} + 2630727x^{20} + \dots$$



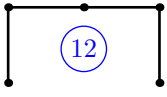
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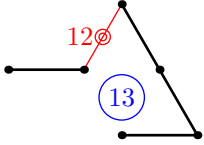
$$A(x)^7 / (1 - A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



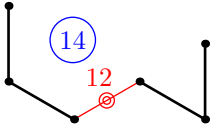
$$(A(x)^8/(1-A(x))^3 - (1+A(x))A(x^2)^4/(1-A(x^2))^2)/2 = 5x^9 + 31x^{10} + 163x^{11} + 674x^{12} + 2566x^{13} + 8953x^{14} + 29807x^{15} + 95015x^{16} + 294172x^{17} + 887724x^{18} + 2627478x^{19} + 7647503x^{20} + \dots$$



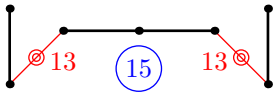
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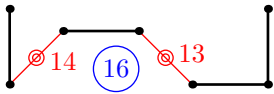
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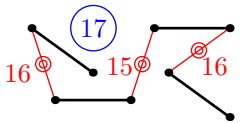
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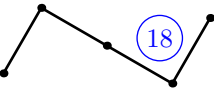
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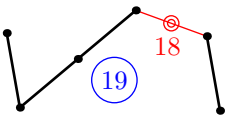
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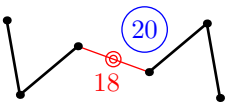
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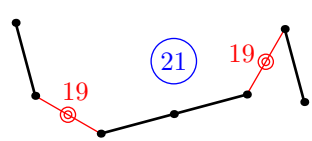
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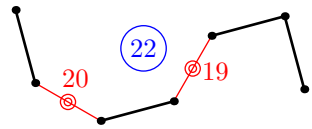
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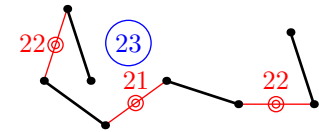
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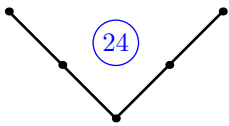
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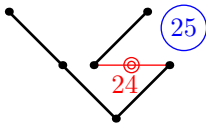
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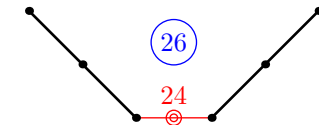
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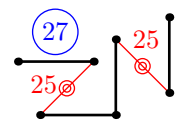
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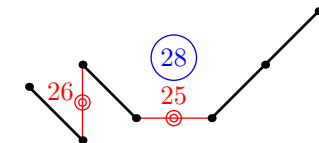
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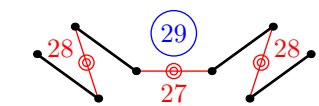
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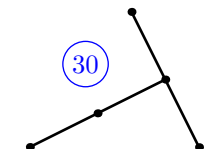
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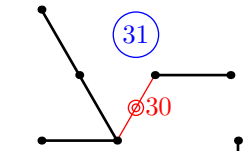
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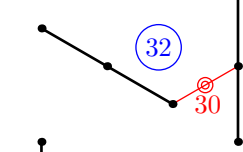
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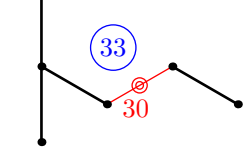
$$A(x)^3 (A(x)^2 - A(x^2)) / 2 = x^6 + 4x^7 + 12x^8 + 33x^9 + 84x^{10} + 208x^{11} + 508x^{12} + 1229x^{13} + 2957x^{14} + 7097x^{15} + 17011x^{16} + 40757x^{17} + 97668x^{18} + 234160x^{19} + 561796x^{20} + \dots$$



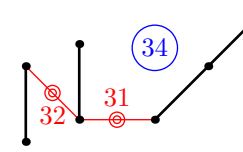
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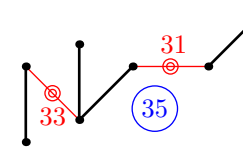
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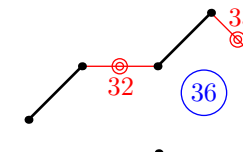
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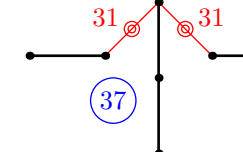
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



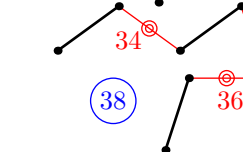
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



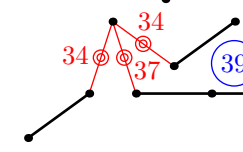
$$A(x)^5(A(x)^2 - A(x^2))/(2(1-A(x))^2) = x^8 + 8x^9 + 40x^{10} + 163x^{11} + 588x^{12} + 1960x^{13} + 6189x^{14} + 18793x^{15} + 55415x^{16} + 159748x^{17} + 452369x^{18} + 1262741x^{19} + 3483665x^{20} + \dots$$



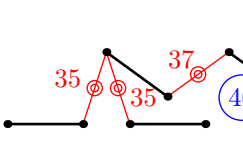
$$A(x)^3(A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2)))/2 = 3x^8 + 19x^9 + 86x^{10} + 325x^{11} + 1109x^{12} + 3537x^{13} + 10783x^{14} + 31803x^{15} + 91526x^{16} + 258445x^{17} + 718977x^{18} + 1976223x^{19} + 5378889x^{20} + \dots$$



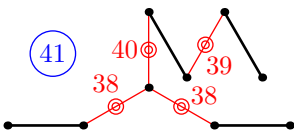
$$A(x)^8/(1-A(x))^3 = x^8 + 11x^9 + 69x^{10} + 333x^{11} + 1379x^{12} + 5164x^{13} + 18023x^{14} + 59743x^{15} + 190444x^{16} + 588844x^{17} + 1776900x^{18} + 5256918x^{19} + 15300266x^{20} + \dots$$



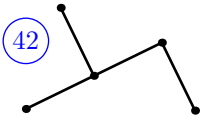
$$A(x)^4(A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2)))/(2(1-A(x))) = 3x^9 + 25x^{10} + 136x^{11} + 600x^{12} + 2334x^{13} + 8347x^{14} + 28133x^{15} + 90743x^{16} + 283008x^{17} + 859575x^{18} + 2555794x^{19} + 7467918x^{20} + \dots$$



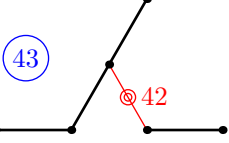
$$A(x)^4(A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2)))/(2(1-A(x))) = 3x^9 + 25x^{10} + 136x^{11} + 600x^{12} + 2334x^{13} + 8347x^{14} + 28133x^{15} + 90743x^{16} + 283008x^{17} + 859575x^{18} + 2555794x^{19} + 7467918x^{20} + \dots$$



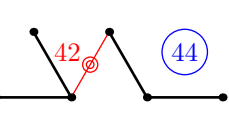
$$A(x)^5 (A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2))) / (2(1-A(x))^2) = 3x^{10} + 31x^{11} + 198x^{12} + 999x^{13} + 4363x^{14} + 17277x^{15} + 63754x^{16} + 223090x^{17} + 749072x^{18} + 2433736x^{19} + 7698294x^{20} + \dots$$



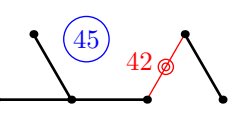
$$A(x)^5 = x^5 + 5x^6 + 15x^7 + 40x^8 + 100x^9 + 241x^{10} + 575x^{11} + 1365x^{12} + 3230x^{13} + 7640x^{14} + 18086x^{15} + 42865x^{16} + 101745x^{17} + 241895x^{18} + 576030x^{19} + 1373937x^{20} + \dots$$



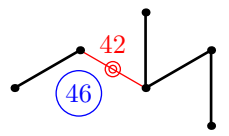
$$A(x)^6/(1-A(x)) = x^6 + 7x^7 + 29x^8 + 99x^9 + 305x^{10} + 882x^{11} + 2454x^{12} + 6656x^{13} + 17731x^{14} + 46625x^{15} + 121435x^{16} + 313984x^{17} + 807281x^{18} + 2066413x^{19} + 5270732x^{20} + \dots$$



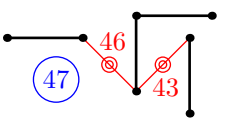
$$A(x)^6/(1-A(x)) = x^6 + 7x^7 + 29x^8 + 99x^9 + 305x^{10} + 882x^{11} + 2454x^{12} + 6656x^{13} + 17731x^{14} + 46625x^{15} + 121435x^{16} + 313984x^{17} + 807281x^{18} + 2066413x^{19} + 5270732x^{20} + \dots$$



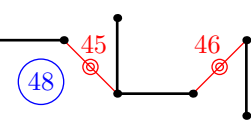
$$A(x)^6/(1-A(x)) = x^6 + 7x^7 + 29x^8 + 99x^9 + 305x^{10} + 882x^{11} + 2454x^{12} + 6656x^{13} + 17731x^{14} + 46625x^{15} + 121435x^{16} + 313984x^{17} + 807281x^{18} + 2066413x^{19} + 5270732x^{20} + \dots$$



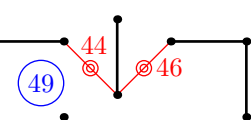
$$A(x)^6/(1-A(x)) = x^6 + 7x^7 + 29x^8 + 99x^9 + 305x^{10} + 882x^{11} + 2454x^{12} + 6656x^{13} + 17731x^{14} + 46625x^{15} + 121435x^{16} + 313984x^{17} + 807281x^{18} + 2066413x^{19} + 5270732x^{20} + \dots$$



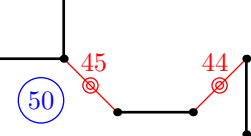
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



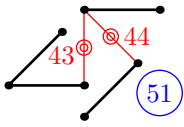
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



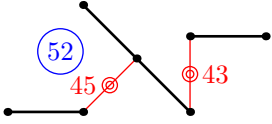
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



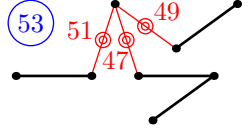
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



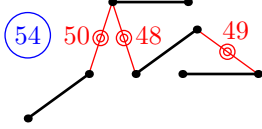
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



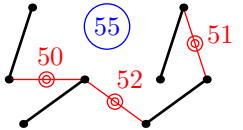
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



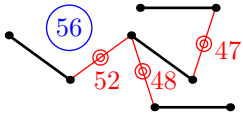
$$A(x)^8/(1-A(x))^3 = x^8 + 11x^9 + 69x^{10} + 333x^{11} + 1379x^{12} + 5164x^{13} + 18023x^{14} + 59743x^{15} + 190444x^{16} + 588844x^{17} + 1776900x^{18} + 5256918x^{19} + 15300266x^{20} + \dots$$



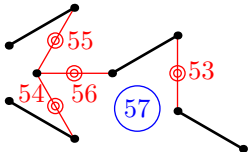
$$A(x)^8/(1-A(x))^3 = x^8 + 11x^9 + 69x^{10} + 333x^{11} + 1379x^{12} + 5164x^{13} + 18023x^{14} + 59743x^{15} + 190444x^{16} + 588844x^{17} + 1776900x^{18} + 5256918x^{19} + 15300266x^{20} + \dots$$



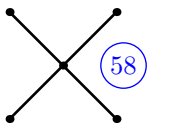
$$A(x)^8/(1-A(x))^3 = x^8 + 11x^9 + 69x^{10} + 333x^{11} + 1379x^{12} + 5164x^{13} + 18023x^{14} + 59743x^{15} + 190444x^{16} + 588844x^{17} + 1776900x^{18} + 5256918x^{19} + 15300266x^{20} + \dots$$



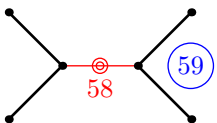
$$A(x)^8/(1-A(x))^3 = x^8 + 11x^9 + 69x^{10} + 333x^{11} + 1379x^{12} + 5164x^{13} + 18023x^{14} + 59743x^{15} + 190444x^{16} + 588844x^{17} + 1776900x^{18} + 5256918x^{19} + 15300266x^{20} + \dots$$



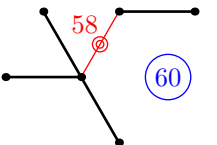
$$A(x)^9/(1-A(x))^4 = x^9 + 13x^{10} + 95x^{11} + 524x^{12} + 2440x^{13} + 10141x^{14} + 38857x^{15} + 140119x^{16} + 482148x^{17} + 1598636x^{18} + 5143810x^{19} + 16147023x^{20} + \dots$$



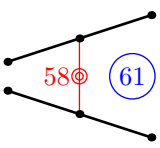
$$A(x) \left((A(x)^2 - A(x^2))^2 / 8 - (A(x^2)^2 - A(x^4)) / 4 \right) = x^8 + 4x^9 + 12x^{10} + 32x^{11} + 84x^{12} + 212x^{13} + 528x^{14} + 1298x^{15} + 3175x^{16} + 7725x^{17} + 18756x^{18} + 45448x^{19} + 110046x^{20} + \dots$$



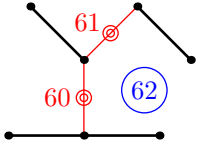
$$A(x)^2 (A(x)^4 - A(x^2)^2) / (4(1-A(x))) - (1+A(x))(A(x^2)^3 - A(x^2)A(x^4)) / (2(1-A(x^2))) = x^7 + 4x^8 + 18x^9 + 58x^{10} + 182x^{11} + 516x^{12} + 1448x^{13} + 3903x^{14} + 10432x^{15} + 27379x^{16} + 71424x^{17} + 184604x^{18} + 475096x^{19} + 1216379x^{20} + \dots$$



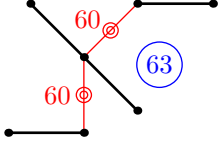
$$A(x)^4 (A(x)^2 - A(x^2)) / (2(1-A(x))) = x^7 + 6x^8 + 24x^9 + 82x^{10} + 254x^{11} + 742x^{12} + 2088x^{13} + 5725x^{14} + 15406x^{15} + 40891x^{16} + 107408x^{17} + 279860x^{18} + 724584x^{19} + 1866521x^{20} + \dots$$



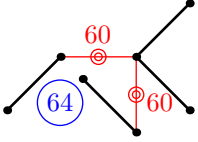
$$A(x)^2 (A(x)^2 - A(x^2))^2 / (8(1-A(x))) - A(x^2)(1+A(x)) (A(x^2)^2 - A(x^4)) / (4(1-A(x^2))) = 2x^9 + 8x^{10} + 32x^{11} + 99x^{12} + 305x^{13} + 866x^{14} + 2433x^{15} + 6609x^{16} + 17791x^{17} + 47119x^{18} + 123942x^{19} + 323139x^{20} + \dots$$



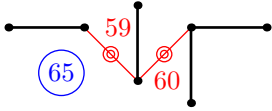
$$A(x)^5 (A(x)^2 - A(x^2)) / (2(1-A(x))^2) = x^8 + 8x^9 + 40x^{10} + 163x^{11} + 588x^{12} + 1960x^{13} + 6189x^{14} + 18793x^{15} + 55415x^{16} + 159748x^{17} + 452369x^{18} + 1262741x^{19} + 3483665x^{20} + \dots$$



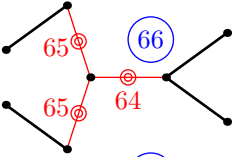
$$A(x) (A(x)^2 - A(x^2)) (A(x)^4 / (1-A(x))^2 - A(x^2)^2 / (1-A(x^2))) / 4 = 3x^9 + 16x^{10} + 73x^{11} + 271x^{12} + 930x^{13} + 2973x^{14} + 9128x^{15} + 27080x^{16} + 78465x^{17} + 222927x^{18} + 623955x^{19} + 1724675x^{20} + \dots$$



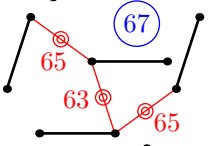
$$A(x) (A(x)^6 / (1-A(x))^2 - A(x^2)^3 / (1-A(x^2))) / 2 = 4x^8 + 21x^9 + 94x^{10} + 342x^{11} + 1155x^{12} + 3637x^{13} + 11026x^{14} + 32340x^{15} + 92781x^{16} + 261263x^{17} + 725492x^{18} + 1991054x^{19} + 5413204x^{20} + \dots$$



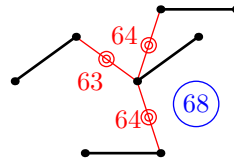
$$A(x)^7 / (1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



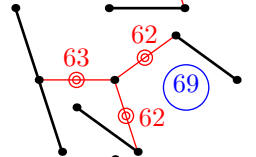
$$A(x)^2 (A(x)^6 / (1-A(x))^2 - A(x^2)^3 / (1-A(x^2))) / (2(1-A(x))) = 4x^9 + 29x^{10} + 152x^{11} + 650x^{12} + 2484x^{13} + 8765x^{14} + 29272x^{15} + 93754x^{16} + 290868x^{17} + 879808x^{18} + 2607508x^{19} + 7599176x^{20} + \dots$$



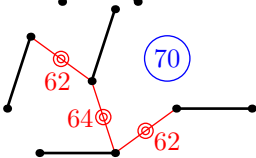
$$(A(x)^8 / (1-A(x))^3 - (1+A(x))A(x^2)^4 / (1-A(x^2))^2) / 2 = 5x^9 + 31x^{10} + 163x^{11} + 674x^{12} + 2566x^{13} + 8953x^{14} + 29807x^{15} + 95015x^{16} + 294172x^{17} + 887724x^{18} + 2627478x^{19} + 7647503x^{20} + \dots$$



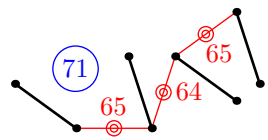
$$A(x)^4 (A(x)^4 / (1-A(x))^2 - A(x^2)^2 / (1-A(x^2))) / (2(1-A(x))) = 3x^9 + 25x^{10} + 136x^{11} + 600x^{12} + 2334x^{13} + 8347x^{14} + 28133x^{15} + 90743x^{16} + 283008x^{17} + 859575x^{18} + 2555794x^{19} + 7467918x^{20} + \dots$$



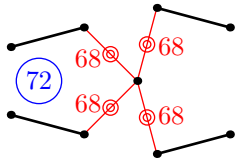
$$A(x)^2 (A(x)^2 - A(x^2)) (A(x)^4 / (1-A(x))^2 - A(x^2)^2 / (1-A(x^2))) / (4(1-A(x))) = 3x^{10} + 22x^{11} + 117x^{12} + 508x^{13} + 1968x^{14} + 7032x^{15} + 23750x^{16} + 76844x^{17} + 240594x^{18} + 733789x^{19} + 2191144x^{20} + \dots$$



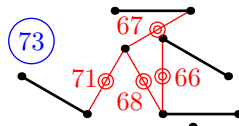
$$(A(x)^8 / (1-A(x))^3 - (1+A(x))A(x^2)^4 / (1-A(x^2))^2) / 2 = 5x^9 + 31x^{10} + 163x^{11} + 674x^{12} + 2566x^{13} + 8953x^{14} + 29807x^{15} + 95015x^{16} + 294172x^{17} + 887724x^{18} + 2627478x^{19} + 7647503x^{20} + \dots$$



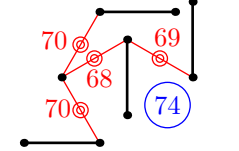
$$(A(x)^8/(1-A(x))^3 - (1+A(x))A(x^2)^4/(1-A(x^2))^2) / 2 = 5x^9 + 31x^{10} + 163x^{11} + 674x^{12} + 2566x^{13} + 8953x^{14} + 29807x^{15} + 95015x^{16} + 294172x^{17} + 887724x^{18} + 2627478x^{19} + 7647503x^{20} + \dots$$



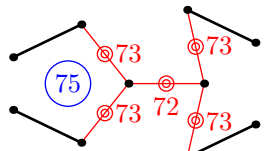
$$A(x) \left((A(x)^4/(1-A(x))^2 - A(x^2)^2/((1-A(x))^2) / 8 - (A(x^2)^4/(1-A(x^2))^2) / (1-A(x^2))^2 - A(x^4)^2/(1-A(x^4))) / 4 \right) = 3x^{11} + 33x^{12} + 192x^{13} + 911x^{14} + 3769x^{15} + 14347x^{16} + 51231x^{17} + 174643x^{18} + 573719x^{19} + 1830505x^{20} + \dots$$



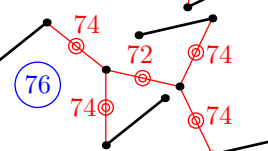
$$A(x)^9/(1-A(x))^4 = x^9 + 13x^{10} + 95x^{11} + 524x^{12} + 2440x^{13} + 10141x^{14} + 38857x^{15} + 140119x^{16} + 482148x^{17} + 1598636x^{18} + 5143810x^{19} + 16147023x^{20} + \dots$$



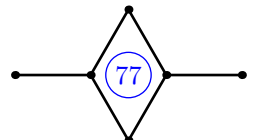
$$A(x)^5 (A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2))) / (2(1-A(x))^2) = 3x^{10} + 31x^{11} + 198x^{12} + 999x^{13} + 4363x^{14} + 17277x^{15} + 63754x^{16} + 223090x^{17} + 749072x^{18} + 2433736x^{19} + 7698294x^{20} + \dots$$



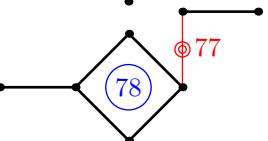
$$A(x)^2 (A(x)^8/(1-A(x))^4 - A(x^2)^4/(1-A(x^2))^2) / (4(1-A(x))) - (1+A(x)) A(x^2) (A(x^2)^4/(1-A(x^2))^2 - A(x^4)^2/(1-A(x^4))) / (2(1-A(x^2))) = 3x^{11} + 25x^{12} + 182x^{13} + 956x^{14} + 4484x^{15} + 18787x^{16} + 73684x^{17} + 272542x^{18} + 966282x^{19} + 3303587x^{20} + \dots$$



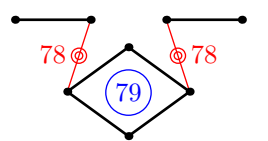
$$A(x)^2 (A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2)))^2 / (8(1-A(x))) - A(x^2) (1+A(x)) (A(x^2)^4/(1-A(x^2))^2 - A(x^4)^2/(1-A(x^4))) / (4(1-A(x^2))) = 3x^{12} + 42x^{13} + 276x^{14} + 1485x^{15} + 6784x^{16} + 28292x^{17} + 109421x^{18} + 401332x^{19} + 1408698x^{20} + \dots$$



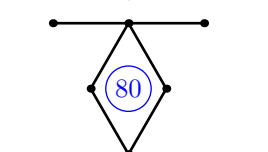
$$(A(x)^2 - A(x^2)) (A(x)^4 - A(x^2)^2) / 4 = 2x^8 + 6x^9 + 22x^{10} + 60x^{11} + 166x^{12} + 426x^{13} + 1094x^{14} + 2732x^{15} + 6808x^{16} + 16780x^{17} + 41276x^{18} + 101060x^{19} + 247132x^{20} + \dots$$



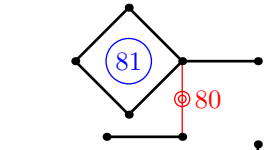
$$A(x)^5 (A(x)^2 - A(x^2)) / (2(1-A(x))) = x^8 + 7x^9 + 31x^{10} + 114x^{11} + 375x^{12} + 1150x^{13} + 3368x^{14} + 9550x^{15} + 26447x^{16} + 71957x^{17} + 193142x^{18} + 512934x^{19} + 1350724x^{20} + \dots$$



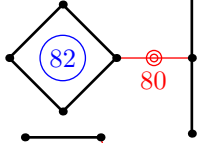
$$(A(x)^2 - A(x^2)) (A(x)^6/(1-A(x))^2 - A(x^2)^3/(1-A(x^2))) / 4 = 4x^{10} + 21x^{11} + 102x^{12} + 388x^{13} + 1384x^{14} + 4544x^{15} + 14346x^{16} + 43541x^{17} + 128919x^{18} + 373231x^{19} + 1062971x^{20} + \dots$$



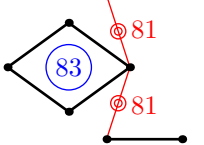
$$A(x)^2 (A(x)^2 - A(x^2))^2 / 4 = x^8 + 4x^9 + 14x^{10} + 42x^{11} + 117x^{12} + 312x^{13} + 811x^{14} + 2068x^{15} + 5210x^{16} + 13010x^{17} + 32289x^{18} + 79774x^{19} + 196463x^{20} + \dots$$



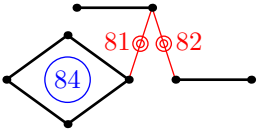
$$A(x)^5 (A(x)^2 - A(x^2)) / (2(1-A(x))) = x^8 + 7x^9 + 31x^{10} + 114x^{11} + 375x^{12} + 1150x^{13} + 3368x^{14} + 9550x^{15} + 26447x^{16} + 71957x^{17} + 193142x^{18} + 512934x^{19} + 1350724x^{20} + \dots$$



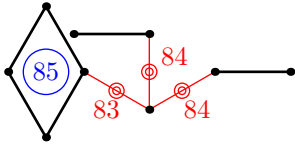
$$A(x)^3 (A(x)^2 - A(x^2))^2 / (4(1-A(x))) = x^9 + 6x^{10} + 26x^{11} + 95x^{12} + 313x^{13} + 966x^{14} + 2852x^{15} + 8155x^{16} + 22773x^{17} + 62458x^{18} + 168912x^{19} + 451747x^{20} + \dots$$



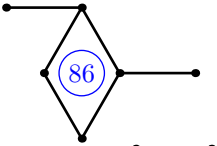
$$A(x)^2 (A(x)^2 - A(x^2)) (A(x)^4 / (1-A(x))^2 - A(x^2)^2 / (1-A(x^2))) / 4 = 3x^{10} + 19x^{11} + 92x^{12} + 366x^{13} + 1315x^{14} + 4386x^{15} + 13924x^{16} + 42559x^{17} + 126467x^{18} + 367475x^{19} + 1049009x^{20} + \dots$$



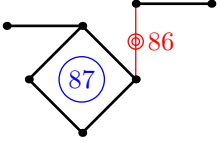
$$A(x)^6 (A(x)^2 - A(x^2)) / (2(1-A(x))^2) = x^9 + 9x^{10} + 49x^{11} + 213x^{12} + 810x^{13} + 2821x^{14} + 9243x^{15} + 28968x^{16} + 87791x^{17} + 259227x^{18} + 749807x^{19} + 2132941x^{20} + \dots$$



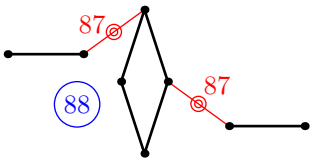
$$A(x)^3 (A(x)^2 - A(x^2)) (A(x)^4 / (1-A(x))^2 - A(x^2)^2 / (1-A(x^2))) / (4(1-A(x))) = 3x^{11} + 25x^{12} + 142x^{13} + 653x^{14} + 2646x^{15} + 9826x^{16} + 34285x^{17} + 114127x^{18} + 366314x^{19} + 1142135x^{20} + \dots$$



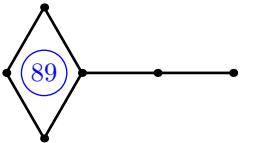
$$(A(x)^6 - A(x^2)^3) / 2 = 3x^7 + 9x^8 + 31x^9 + 81x^{10} + 216x^{11} + 535x^{12} + 1338x^{13} + 3261x^{14} + 7968x^{15} + 19296x^{16} + 46773x^{17} + 113024x^{18} + 273282x^{19} + 660201x^{20} + \dots$$



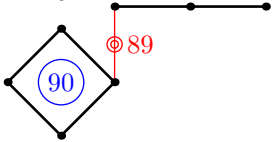
$$A(x)^7 / (1-A(x)) = x^7 + 8x^8 + 37x^9 + 137x^{10} + 450x^{11} + 1371x^{12} + 3980x^{13} + 11182x^{14} + 30689x^{15} + 82786x^{16} + 220438x^{17} + 581109x^{18} + 1519849x^{19} + 3950057x^{20} + \dots$$



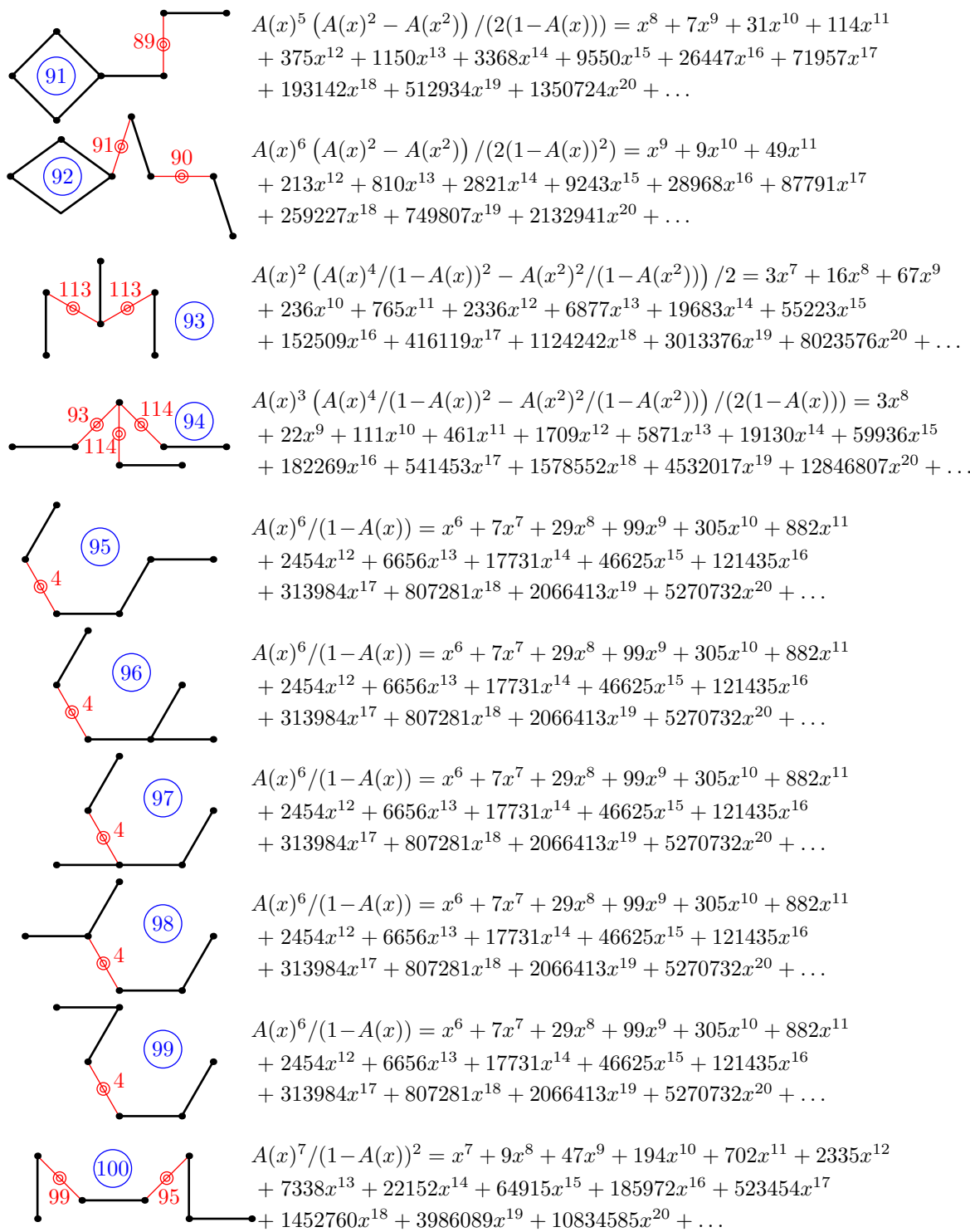
$$(A(x)^8 / (1-A(x))^2 - A(x^2)^4 / (1-A(x^2))) / 2 = 5x^9 + 26x^{10} + 126x^{11} + 474x^{12} + 1679x^{13} + 5462x^{14} + 17113x^{15} + 51531x^{16} + 151508x^{17} + 435664x^{18} + 1233120x^{19} + 3441850x^{20} + \dots$$

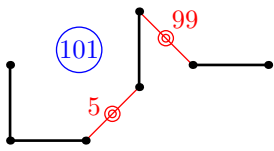


$$A(x)^4 (A(x)^2 - A(x^2)) / 2 = x^7 + 5x^8 + 17x^9 + 51x^{10} + 140x^{11} + 367x^{12} + 938x^{13} + 2357x^{14} + 5856x^{15} + 14444x^{16} + 35451x^{17} + 86718x^{18} + 211650x^{19} + 515797x^{20} + \dots$$

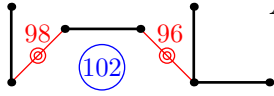


$$A(x)^5 (A(x)^2 - A(x^2)) / (2(1-A(x))) = x^8 + 7x^9 + 31x^{10} + 114x^{11} + 375x^{12} + 1150x^{13} + 3368x^{14} + 9550x^{15} + 26447x^{16} + 71957x^{17} + 193142x^{18} + 512934x^{19} + 1350724x^{20} + \dots$$

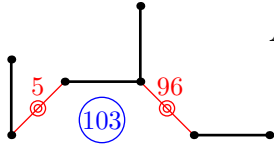




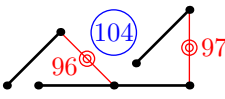
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



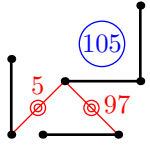
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



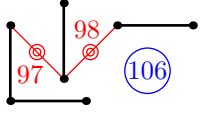
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



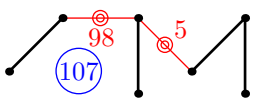
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



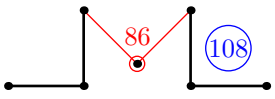
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



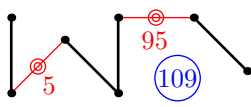
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



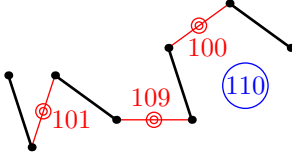
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



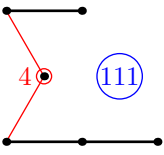
$$(A(x)^7/(1-A(x)) - (A(x) + A(x^2))A(x^2)^3/(1-A(x^2))) / 2 = 3x^8 + 16x^9 + 63x^{10} + 216x^{11} + 665x^{12} + 1958x^{13} + 5518x^{14} + 15227x^{15} + 41126x^{16} + 109755x^{17} + 289505x^{18} + 757934x^{19} + 1970526x^{20} + \dots$$



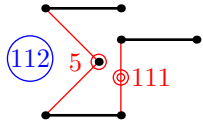
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



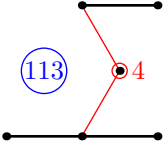
$$A(x)^8/(1-A(x))^3 = x^8 + 11x^9 + 69x^{10} + 333x^{11} + 1379x^{12} + 5164x^{13} + 18023x^{14} + 59743x^{15} + 190444x^{16} + 588844x^{17} + 1776900x^{18} + 5256918x^{19} + 15300266x^{20} + \dots$$



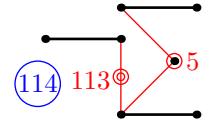
$$A(x)^6/(1-A(x)) = x^6 + 7x^7 + 29x^8 + 99x^9 + 305x^{10} + 882x^{11} \\ + 2454x^{12} + 6656x^{13} + 17731x^{14} + 46625x^{15} + 121435x^{16} \\ + 313984x^{17} + 807281x^{18} + 2066413x^{19} + 5270732x^{20} + \dots$$



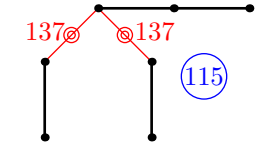
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} \\ + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} \\ + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



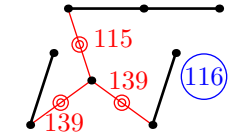
$$A(x)^6/(1-A(x)) = x^6 + 7x^7 + 29x^8 + 99x^9 + 305x^{10} + 882x^{11} \\ + 2454x^{12} + 6656x^{13} + 17731x^{14} + 46625x^{15} + 121435x^{16} \\ + 313984x^{17} + 807281x^{18} + 2066413x^{19} + 5270732x^{20} + \dots$$



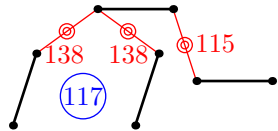
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} \\ + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} \\ + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



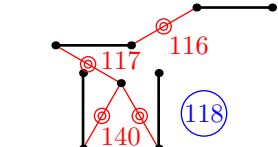
$$A(x)^3 (A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2))) / 2 = 3x^8 + 19x^9 \\ + 86x^{10} + 325x^{11} + 1109x^{12} + 3537x^{13} + 10783x^{14} + 31803x^{15} \\ + 91526x^{16} + 258445x^{17} + 718977x^{18} + 1976223x^{19} + 5378889x^{20} + \dots$$



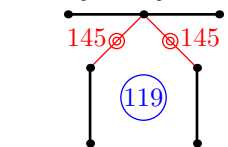
$$A(x)^4 (A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2))) / (2(1-A(x))) = 3x^9 \\ + 25x^{10} + 136x^{11} + 600x^{12} + 2334x^{13} + 8347x^{14} + 28133x^{15} + 90743x^{16} \\ + 283008x^{17} + 859575x^{18} + 2555794x^{19} + 7467918x^{20} + \dots$$



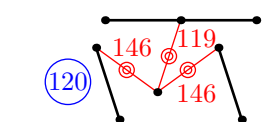
$$A(x)^4 (A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2))) / (2(1-A(x))) = 3x^9 \\ + 25x^{10} + 136x^{11} + 600x^{12} + 2334x^{13} + 8347x^{14} + 28133x^{15} + 90743x^{16} \\ + 283008x^{17} + 859575x^{18} + 2555794x^{19} + 7467918x^{20} + \dots$$



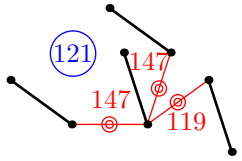
$$A(x)^5 (A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2))) / (2(1-A(x))^2) = 3x^{10} \\ + 31x^{11} + 198x^{12} + 999x^{13} + 4363x^{14} + 17277x^{15} + 63754x^{16} \\ + 223090x^{17} + 749072x^{18} + 2433736x^{19} + 7698294x^{20} + \dots$$



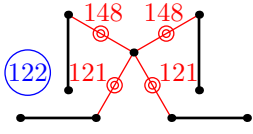
$$A(x) (A(x)^2 - A(x^2)) (A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2))) / 4 = \\ 3x^9 + 16x^{10} + 73x^{11} + 271x^{12} + 930x^{13} + 2973x^{14} + 9128x^{15} + 27080x^{16} \\ + 78465x^{17} + 222927x^{18} + 623955x^{19} + 1724675x^{20} + \dots$$



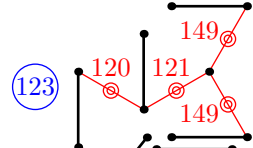
$$A(x)^2 (A(x)^2 - A(x^2)) (A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2))) \\ / (4(1-A(x))) = 3x^{10} + 22x^{11} + 117x^{12} + 508x^{13} + 1968x^{14} + 7032x^{15} \\ + 23750x^{16} + 76844x^{17} + 240594x^{18} + 733789x^{19} + 2191144x^{20} + \dots$$



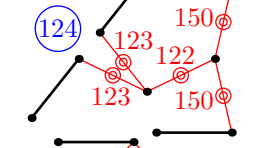
$$A(x)^4 (A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2))) / (2(1-A(x))) = 3x^9 + 25x^{10} + 136x^{11} + 600x^{12} + 2334x^{13} + 8347x^{14} + 28133x^{15} + 90743x^{16} + 283008x^{17} + 859575x^{18} + 2555794x^{19} + 7467918x^{20} + \dots$$



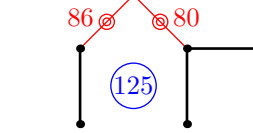
$$A(x) (A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2)))^2 / 4 = 9x^{11} + 69x^{12} + 397x^{13} + 1838x^{14} + 7595x^{15} + 28770x^{16} + 102678x^{17} + 349609x^{18} + 1148274x^{19} + 3662385x^{20} + \dots$$



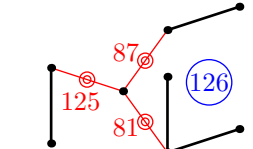
$$A(x)^5 (A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2))) / (2(1-A(x))^2) = 3x^{10} + 31x^{11} + 198x^{12} + 999x^{13} + 4363x^{14} + 17277x^{15} + 63754x^{16} + 223090x^{17} + 749072x^{18} + 2433736x^{19} + 7698294x^{20} + \dots$$



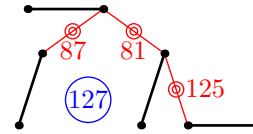
$$A(x)^2 (A(x)^4/(1-A(x))^2 - A(x^2)^2/(1-A(x^2)))^2 / (4(1-A(x))) = 9x^{12} + 87x^{13} + 571x^{14} + 2989x^{15} + 13660x^{16} + 56679x^{17} + 219217x^{18} + 803073x^{19} + 2818822x^{20} + \dots$$



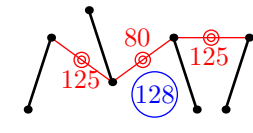
$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$



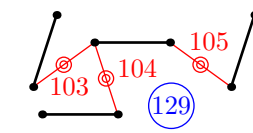
$$A(x)^8/(1-A(x))^3 = x^8 + 11x^9 + 69x^{10} + 333x^{11} + 1379x^{12} + 5164x^{13} + 18023x^{14} + 59743x^{15} + 190444x^{16} + 588844x^{17} + 1776900x^{18} + 5256918x^{19} + 15300266x^{20} + \dots$$



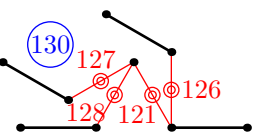
$$A(x)^8/(1-A(x))^3 = x^8 + 11x^9 + 69x^{10} + 333x^{11} + 1379x^{12} + 5164x^{13} + 18023x^{14} + 59743x^{15} + 190444x^{16} + 588844x^{17} + 1776900x^{18} + 5256918x^{19} + 15300266x^{20} + \dots$$



$$(A(x)^8/(1-A(x))^3 - (1+A(x))A(x^2)^4/(1-A(x^2))^2) / 2 = 5x^9 + 31x^{10} + 163x^{11} + 674x^{12} + 2566x^{13} + 8953x^{14} + 29807x^{15} + 95015x^{16} + 294172x^{17} + 887724x^{18} + 2627478x^{19} + 7647503x^{20} + \dots$$



$$A(x)^8/(1-A(x))^3 = x^8 + 11x^9 + 69x^{10} + 333x^{11} + 1379x^{12} + 5164x^{13} + 18023x^{14} + 59743x^{15} + 190444x^{16} + 588844x^{17} + 1776900x^{18} + 5256918x^{19} + 15300266x^{20} + \dots$$



$$A(x)^9/(1-A(x))^4 = x^9 + 13x^{10} + 95x^{11} + 524x^{12} + 2440x^{13} + 10141x^{14} + 38857x^{15} + 140119x^{16} + 482148x^{17} + 1598636x^{18} + 5143810x^{19} + 16147023x^{20} + \dots$$

$$A(x)^9/(1-A(x))^4 = x^9 + 13x^{10} + 95x^{11} + 524x^{12} + 2440x^{13} + 10141x^{14} + 38857x^{15} + 140119x^{16} + 482148x^{17} + 1598636x^{18} + 5143810x^{19} + 16147023x^{20} + \dots$$

$$(A(x)^{10}/(1-A(x))^5 - (1+A(x))A(x^2)^5/(1-A(x^2))^3)/2 = 7x^{11} + 58x^{12} + 383x^{13} + 1978x^{14} + 9114x^{15} + 38002x^{16} + 148310x^{17} + 547585x^{18} + 1938203x^{19} + 6621472x^{20} + \dots$$

$$A(x)^7/(1-A(x))^2 = x^7 + 9x^8 + 47x^9 + 194x^{10} + 702x^{11} + 2335x^{12} + 7338x^{13} + 22152x^{14} + 64915x^{15} + 185972x^{16} + 523454x^{17} + 1452760x^{18} + 3986089x^{19} + 10834585x^{20} + \dots$$

$$A(x)^8/(1-A(x))^3 = x^8 + 11x^9 + 69x^{10} + 333x^{11} + 1379x^{12} + 5164x^{13} + 18023x^{14} + 59743x^{15} + 190444x^{16} + 588844x^{17} + 1776900x^{18} + 5256918x^{19} + 15300266x^{20} + \dots$$

$$A(x)^8/(1-A(x))^3 = x^8 + 11x^9 + 69x^{10} + 333x^{11} + 1379x^{12} + 5164x^{13} + 18023x^{14} + 59743x^{15} + 190444x^{16} + 588844x^{17} + 1776900x^{18} + 5256918x^{19} + 15300266x^{20} + \dots$$

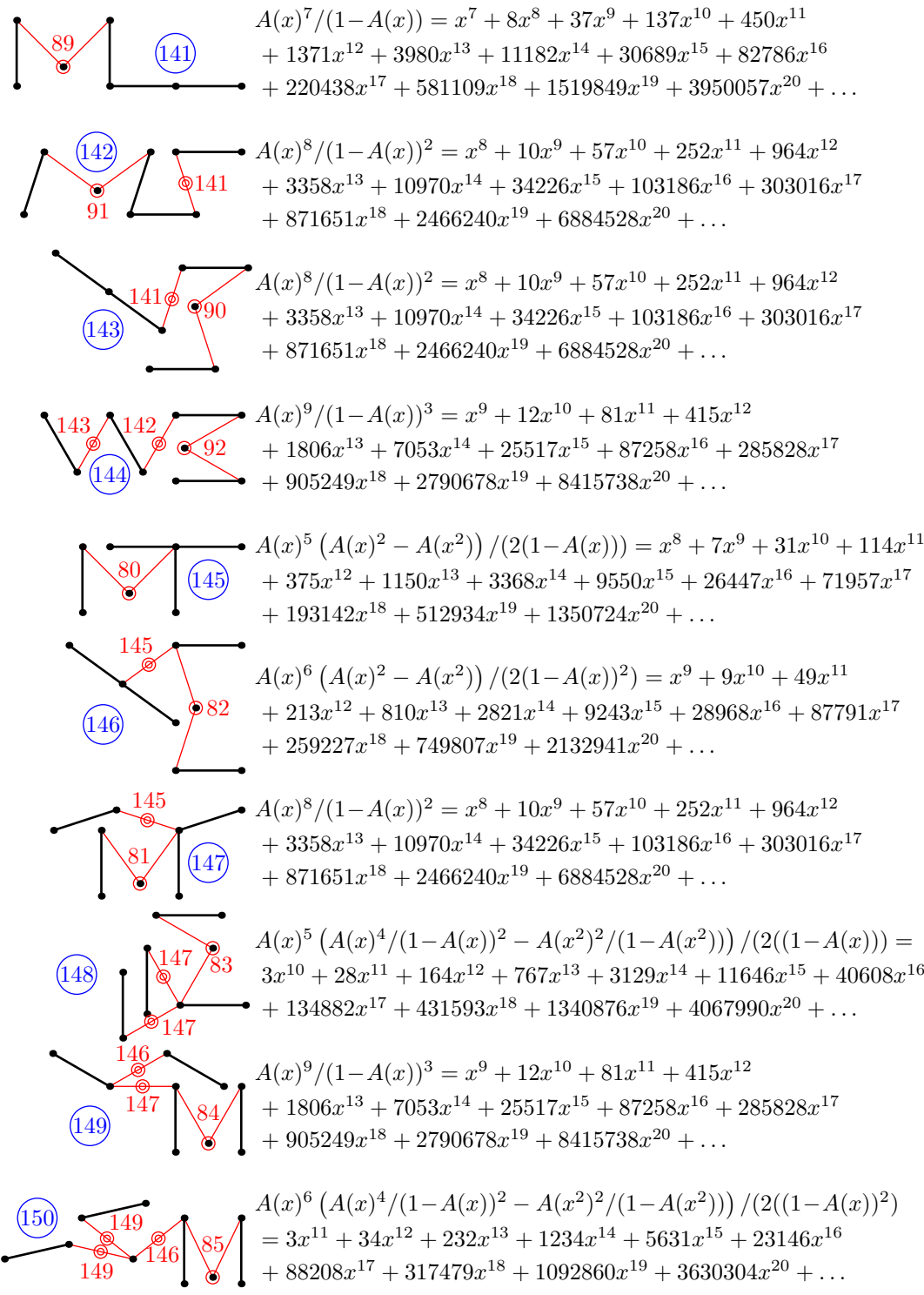
$$A(x)^9/(1-A(x))^4 = x^9 + 13x^{10} + 95x^{11} + 524x^{12} + 2440x^{13} + 10141x^{14} + 38857x^{15} + 140119x^{16} + 482148x^{17} + 1598636x^{18} + 5143810x^{19} + 16147023x^{20} + \dots$$

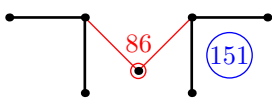
$$A(x)^7/(1-A(x)) = x^7 + 8x^8 + 37x^9 + 137x^{10} + 450x^{11} + 1371x^{12} + 3980x^{13} + 11182x^{14} + 30689x^{15} + 82786x^{16} + 220438x^{17} + 581109x^{18} + 1519849x^{19} + 3950057x^{20} + \dots$$

$$A(x)^8/(1-A(x))^2 = x^8 + 10x^9 + 57x^{10} + 252x^{11} + 964x^{12} + 3358x^{13} + 10970x^{14} + 34226x^{15} + 103186x^{16} + 303016x^{17} + 871651x^{18} + 2466240x^{19} + 6884528x^{20} + \dots$$

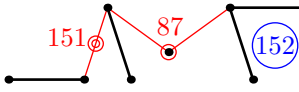
$$A(x)^8/(1-A(x))^2 = x^8 + 10x^9 + 57x^{10} + 252x^{11} + 964x^{12} + 3358x^{13} + 10970x^{14} + 34226x^{15} + 103186x^{16} + 303016x^{17} + 871651x^{18} + 2466240x^{19} + 6884528x^{20} + \dots$$

$$A(x)^9/(1-A(x))^3 = x^9 + 12x^{10} + 81x^{11} + 415x^{12} + 1806x^{13} + 7053x^{14} + 25517x^{15} + 87258x^{16} + 285828x^{17} + 905249x^{18} + 2790678x^{19} + 8415738x^{20} + \dots$$

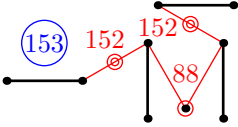




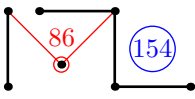
$$(A(x)^7/(1-A(x)) - (A(x) + A(x^2))A(x^2)^3/(1-A(x^2))) / 2 = 3x^8 + 16x^9 + 63x^{10} + 216x^{11} + 665x^{12} + 1958x^{13} + 5518x^{14} + 15227x^{15} + 41126x^{16} + 109755x^{17} + 289505x^{18} + 757934x^{19} + 1970526x^{20} + \dots$$



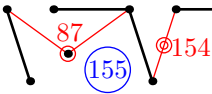
$$A(x)^8/(1-A(x))^2 = x^8 + 10x^9 + 57x^{10} + 252x^{11} + 964x^{12} + 3358x^{13} + 10970x^{14} + 34226x^{15} + 103186x^{16} + 303016x^{17} + 871651x^{18} + 2466240x^{19} + 6884528x^{20} + \dots$$



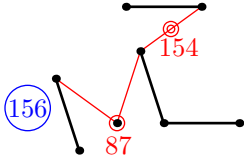
$$(A(x)^9/(1-A(x))^3 - (A(x) + A(x^2))A(x^2)^4/(1-A(x^2))^2) / 2 = 5x^{10} + 37x^{11} + 200x^{12} + 887x^{13} + 3491x^{14} + 12694x^{15} + 43484x^{16} + 142664x^{17} + 452060x^{18} + 1394358x^{19} + 4205653x^{20} + \dots$$



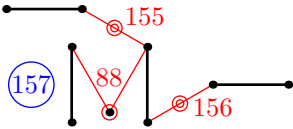
$$A(x)^7/(1-A(x)) = x^7 + 8x^8 + 37x^9 + 137x^{10} + 450x^{11} + 1371x^{12} + 3980x^{13} + 11182x^{14} + 30689x^{15} + 82786x^{16} + 220438x^{17} + 581109x^{18} + 1519849x^{19} + 3950057x^{20} + \dots$$



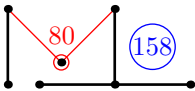
$$A(x)^8/(1-A(x))^2 = x^8 + 10x^9 + 57x^{10} + 252x^{11} + 964x^{12} + 3358x^{13} + 10970x^{14} + 34226x^{15} + 103186x^{16} + 303016x^{17} + 871651x^{18} + 2466240x^{19} + 6884528x^{20} + \dots$$



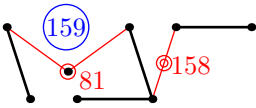
$$A(x)^8/(1-A(x))^2 = x^8 + 10x^9 + 57x^{10} + 252x^{11} + 964x^{12} + 3358x^{13} + 10970x^{14} + 34226x^{15} + 103186x^{16} + 303016x^{17} + 871651x^{18} + 2466240x^{19} + 6884528x^{20} + \dots$$



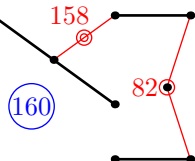
$$A(x)^9/(1-A(x))^3 = x^9 + 12x^{10} + 81x^{11} + 415x^{12} + 1806x^{13} + 7053x^{14} + 25517x^{15} + 87258x^{16} + 285828x^{17} + 905249x^{18} + 2790678x^{19} + 8415738x^{20} + \dots$$



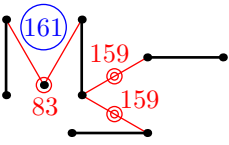
$$A(x)^5 (A(x)^2 - A(x^2)) / (2(1-A(x))) = x^8 + 7x^9 + 31x^{10} + 114x^{11} + 375x^{12} + 1150x^{13} + 3368x^{14} + 9550x^{15} + 26447x^{16} + 71957x^{17} + 193142x^{18} + 512934x^{19} + 1350724x^{20} + \dots$$



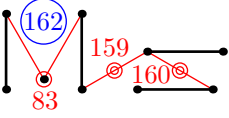
$$A(x)^8/(1-A(x))^2 = x^8 + 10x^9 + 57x^{10} + 252x^{11} + 964x^{12} + 3358x^{13} + 10970x^{14} + 34226x^{15} + 103186x^{16} + 303016x^{17} + 871651x^{18} + 2466240x^{19} + 6884528x^{20} + \dots$$



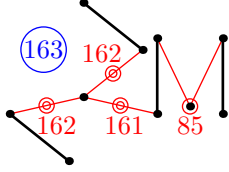
$$A(x)^6 (A(x)^2 - A(x^2)) / (2(1-A(x))^2) = x^9 + 9x^{10} + 49x^{11} + 213x^{12} + 810x^{13} + 2821x^{14} + 9243x^{15} + 28968x^{16} + 87791x^{17} + 259227x^{18} + 749807x^{19} + 2132941x^{20} + \dots$$



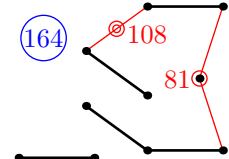
$$A(x)^5 (A(x)^4 / (1-A(x))^2 - A(x^2)^2 / (1-A(x^2))) / (2((1-A(x))) = 3x^{10} + 28x^{11} + 164x^{12} + 767x^{13} + 3129x^{14} + 11646x^{15} + 40608x^{16} + 134882x^{17} + 431593x^{18} + 1340876x^{19} + 4067990x^{20} + \dots$$



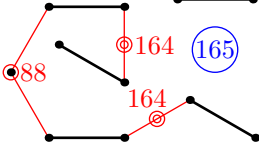
$$A(x)^9 / (1-A(x))^3 = x^9 + 12x^{10} + 81x^{11} + 415x^{12} + 1806x^{13} + 7053x^{14} + 25517x^{15} + 87258x^{16} + 285828x^{17} + 905249x^{18} + 2790678x^{19} + 8415738x^{20} + \dots$$



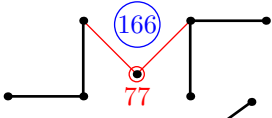
$$A(x)^6 (A(x)^4 / (1-A(x))^2 - A(x^2)^2 / (1-A(x^2))) / (2((1-A(x))^2) = 3x^{11} + 34x^{12} + 232x^{13} + 1234x^{14} + 5631x^{15} + 23146x^{16} + 88208x^{17} + 317479x^{18} + 1092860x^{19} + 3630304x^{20} + \dots$$



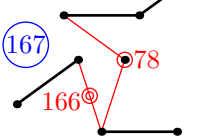
$$A(x)^8 / (1-A(x))^2 = x^8 + 10x^9 + 57x^{10} + 252x^{11} + 964x^{12} + 3358x^{13} + 10970x^{14} + 34226x^{15} + 103186x^{16} + 303016x^{17} + 871651x^{18} + 2466240x^{19} + 6884528x^{20} + \dots$$



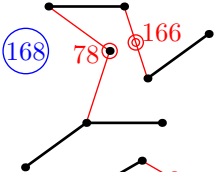
$$(A(x)^9 / (1-A(x))^3 - (A(x) + A(x^2))A(x^2)^4 / (1-A(x^2))^2) / 2 = 5x^{10} + 37x^{11} + 200x^{12} + 887x^{13} + 3491x^{14} + 12694x^{15} + 43484x^{16} + 142664x^{17} + 452060x^{18} + 1394358x^{19} + 4205653x^{20} + \dots$$



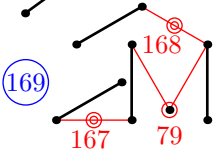
$$A(x)^7 / (1-A(x)) = x^7 + 8x^8 + 37x^9 + 137x^{10} + 450x^{11} + 1371x^{12} + 3980x^{13} + 11182x^{14} + 30689x^{15} + 82786x^{16} + 220438x^{17} + 581109x^{18} + 1519849x^{19} + 3950057x^{20} + \dots$$



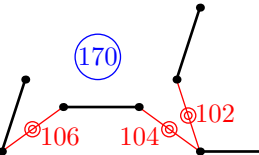
$$A(x)^8 / (1-A(x))^2 = x^8 + 10x^9 + 57x^{10} + 252x^{11} + 964x^{12} + 3358x^{13} + 10970x^{14} + 34226x^{15} + 103186x^{16} + 303016x^{17} + 871651x^{18} + 2466240x^{19} + 6884528x^{20} + \dots$$



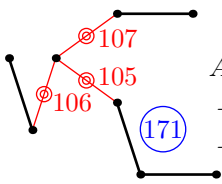
$$A(x)^8 / (1-A(x))^2 = x^8 + 10x^9 + 57x^{10} + 252x^{11} + 964x^{12} + 3358x^{13} + 10970x^{14} + 34226x^{15} + 103186x^{16} + 303016x^{17} + 871651x^{18} + 2466240x^{19} + 6884528x^{20} + \dots$$



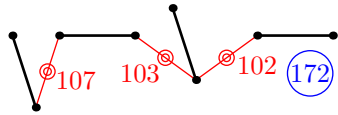
$$A(x)^9 / (1-A(x))^3 = x^9 + 12x^{10} + 81x^{11} + 415x^{12} + 1806x^{13} + 7053x^{14} + 25517x^{15} + 87258x^{16} + 285828x^{17} + 905249x^{18} + 2790678x^{19} + 8415738x^{20} + \dots$$



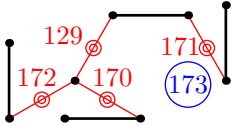
$$A(x)^8 / (1-A(x))^3 = x^8 + 11x^9 + 69x^{10} + 333x^{11} + 1379x^{12} + 5164x^{13} + 18023x^{14} + 59743x^{15} + 190444x^{16} + 588844x^{17} + 1776900x^{18} + 5256918x^{19} + 15300266x^{20} + \dots$$



$$A(x)^8/(1-A(x))^3 = x^8 + 11x^9 + 69x^{10} + 333x^{11} + 1379x^{12} + 5164x^{13} + 18023x^{14} + 59743x^{15} + 190444x^{16} + 588844x^{17} + 1776900x^{18} + 5256918x^{19} + 15300266x^{20} + \dots$$



$$A(x)^8/(1-A(x))^3 = x^8 + 11x^9 + 69x^{10} + 333x^{11} + 1379x^{12} + 5164x^{13} + 18023x^{14} + 59743x^{15} + 190444x^{16} + 588844x^{17} + 1776900x^{18} + 5256918x^{19} + 15300266x^{20} + \dots$$



$$A(x)^9/(1-A(x))^4 = x^9 + 13x^{10} + 95x^{11} + 524x^{12} + 2440x^{13} + 10141x^{14} + 38857x^{15} + 140119x^{16} + 482148x^{17} + 1598636x^{18} + 5143810x^{19} + 16147023x^{20} + \dots$$