In this document we prove that the limit of the ratio of successive terms of the sequence A347913, if it exists, is ≥ 2 . We do this by providing a construction for C(n+1) ((n+1)th Catalan number) multisets where there are 2n elements of the multisets. To do this, we first provide a construction for a desired position called H(n), then we show how to get C(n+1) multisets from H(n). This works because the limiting ratio of the Catalan numbers is 4.

Proof: H(n) is the multiset where all the integers with absolute value less than or equal to n-1 and the same parity as n-1 appear twice. The cardinality of H(n) is 2n.

The construction for H(n) is inductive. To construct H(n), you construct H(n-1) and then you apply the final step X(n).

The construction for X(n) is also inductive. You split all the duplicates simultaneously, then you apply X(n-1), then you split all the duplicates simultaneously again. It is easy to prove that these steps result in H(n).

There are n duplicates in H(n). If you split two adjacent duplicates, you get a new duplicate which you can split. Therefore, the duplicate split patterns come in a zero-one array. A zero-one array is defined as a triangular array of zeros and ones such that if there is a one in a square which is not on the diagonal, , then there has to be a one to the right of it and on top of it. Here is an example of such an array:

			1
		1	1
	1	1	0
0	0	0	0

We can construct a bijection between these arrays and Dyck paths to prove that these arrays are counted by the Catalan numbers. You start two squares to the left of the bottom-left corner of the array. Then, you walk right whenever you see a one or a blank space. Otherwise, you walk up. When you reach the topmost blank space, one square above the top-right corner, you walk up. For the example give above, the Dyck path is:

		•

Now given the Dyck path, how do you construct the zero-one array? This is our example Dyck path:

			•
			•
	-	•	

Starting two squares to the right of the bottom-left corner, you fill in a zero if the path goes above the square and a one if the path goes through or below the square. This way, you get the original array. The array that you get from this path is:

			1
		1	1
	1	1	0
0	0	0	0

We got our original array back. This argument proves that the number of positions you can reach in Linee starting with H(n) is the (n+1)th Catalan number. Therefore, the limiting ratio of A347913 is at least 2.