

In this document we prove that the limit of the ratio of successive terms of the sequence A347913, if it exists, is ≤ 6.75 .

Proof: If we can get to multiset A from the initial multiset via splitting, then we can get from multiset A to the initial multiset via “reverse splitting.”

We can define an “island” as a sub-multiset of a multiset of integers such that every element in the rest of the multiset has difference at least 3 with every element of the island. No two islands can combine into one island via reverse splitting because reverse splitting only combines elements with a difference of 2 and the initial multiset consists of only one island. Therefore, the final multiset consists of only one island.

The sum of all elements is 0 because the initial sum is 0 and every split preserves the sum. If there was an element n or greater, then the set would have a sum of at least n or have multiple islands. If there was an element $-n$ or less, then the set would have sum at most $-n$ or have multiple islands. Therefore, there are only $2n-1$ possible distinct elements for the multiset.

Using the formula for the multichoose function, we get that there are $\binom{3n-2}{n}$ ways to satisfy the bound but not necessarily satisfy any other constraints. The limiting ratio between two consecutive terms of this sequence is $27/4 = 6.75$. Therefore, the limit of the ratio of 2 consecutive terms of A347913, if it exists, must be at most 6.75.