Cinquante signes

La tour d'échecs et l'essuie-glace

juin 27, 2021

Posté il y a une heure sur *Math Fun* – avec une réponse rapide de **Neil Bickford** :

Hello Math Fun,
we want this rook to visit exactly once
all terms of S, S being the lexicographically
earliest sequence of distinct nonnegative terms.
The rook starts to move Left 0 steps.
At every step we will now visit an unvisited

term of S, a bit like a windshield wiper moves:

The rook moves Right 2 steps.
Then Left 1 step.
R 5 steps
L 3
R 6
L 4
R 7 — L 8 — R 12 — L 9 — R 11 — L 10 — R 15 — L 13 — R 17 — L 16 …
The seq S = 0,2,1,5,3,6,4,7,8,12,9,11,10,15,13,17,16,…
is not in the OEIS (modulo errors from the author).
Best,
É.

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math-fun mailing list -- math-fun@mailman.xmission.com
To unsubscribe send an email to math-fun-leave@mailman.xmission.com

Neil Bickford was quick to confirm the above terms, extend S and send a diagram:

I had some extra time on the train today, so I wrote a short program to compute this sequence.

Assuming I've got everything correct, the first 101 terms of the sequence (not counting the initial 0) are

{2,1,5,3,6,4,7,8,12,9,11,10,15,13,17,14,16,18,22,19,21,20,25,23,26,24,28,27,30,29,32,31,35,33,36,34,37,38,42,39,43,40,44,41,45,47,46,48,55,49,51,50,53,52,57,54,56,58,62,59,63,60,64,61,65,67,66,68,74,69,72,70,75,71,73,76,79,77,80,78,83,81,85,82,84,86,90,87,91,88,92,89,93,95,94,96,102,97,99,98,103}

and the partial sums/visited cells (including the initial 0) up to that point are

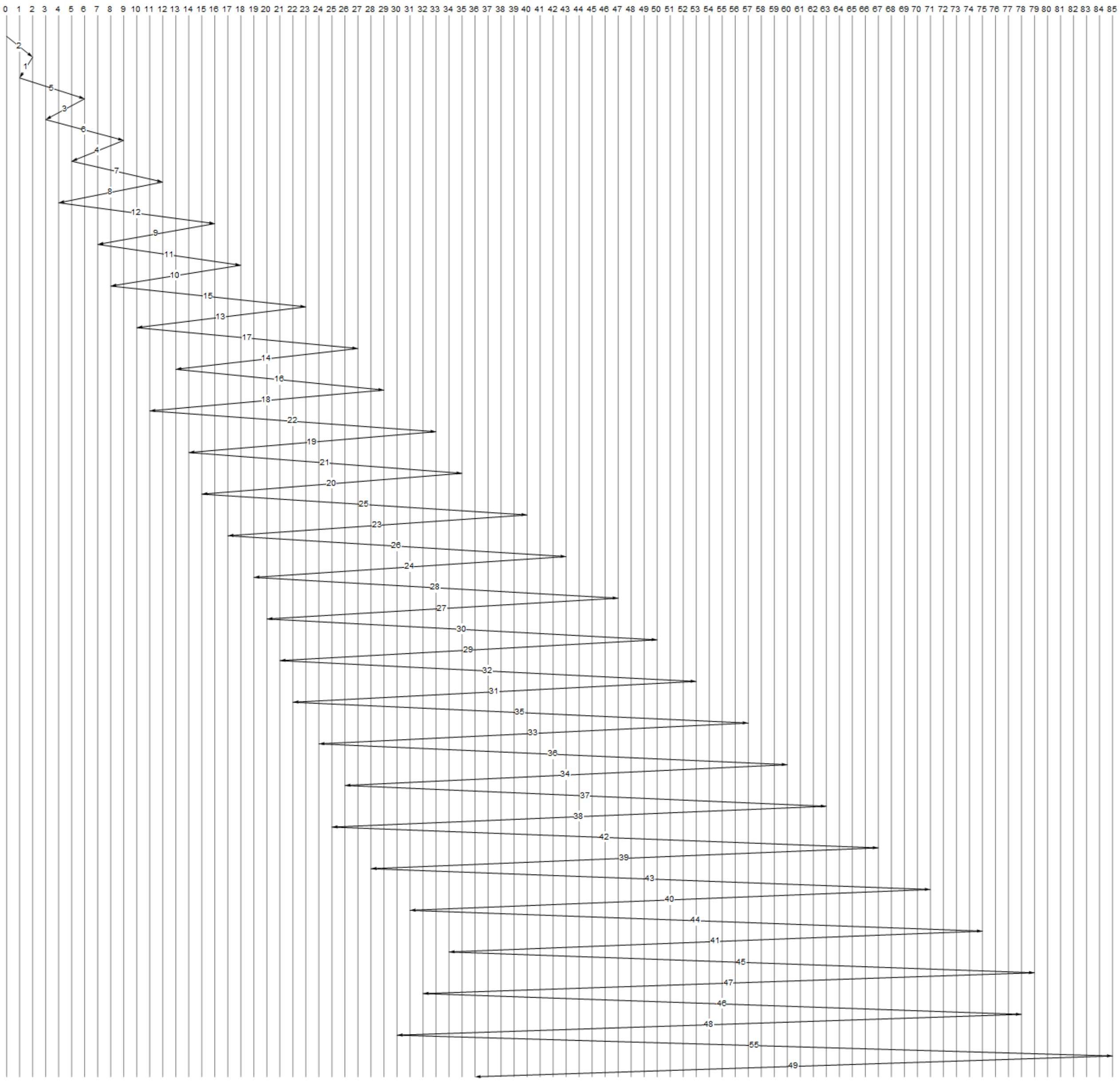
{0,2,1,6,3,9,5,12,4,16,7,18,8,23,10,27,13,29,11,33,14,35,15,40,17,43,19,47,20,50,21,53,22,57,24,60,26,63,25,67,28,71,31,75,34,79,32,78,30,85,36,87,37,90,38,95,41,97,39,101,42,105,45,109,48,113,46

,112,44,118,49,121,51,126,55,128,52,131,54,134,56,139,58,143,61,145,59,149,62,153,65,157,68,161,66,160,64,166,69,168,70,173}

Here's a quick diagram showing how the rook moves over the first 50 segments: https://neilbickford.com/files/ReversingPaths/reversePaths.png

It never quite seems to settle down into a regular pattern so far as I can tell.

Finally, here's a very basic C++ program to do depth-first search for this sequence - in case the formatting gets garbled, it's also available at https://neilbickford.com/files/ReversingPaths/main.cpp . For the above sequence, I had it compute 110 segments, then ignored the last few. Even then, I don't really have a proof that the above terms are correct, and that it won't eventually backtrack over them (...)



Many thanks and bravo, **Neil**! (this is now https://oeis.org/A345967)

Update, July 8th, on Math-Fun, by **Fred Lunnon**:

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Angelini's Rook-Wiper Sequences
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After repeated microscopic inspection of Éric Angelini's
introduction
and algorithmic illustration, some reverse engineering of Neil
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Bickford's heroic C++ program, and multiple bouts of "guess-the-generator" spent staring at its output, I hope finally to have dispelled my previous incomprehension concerning this tantalising sequential chimaera --- I get it, and it's a peach!

(#0) **Motivation:**
The scenario suggesting this topic involves a rook starting from one corner of an infinite chessboard, then leaping in alternate directions along one edge, after the fashion of some (severely dysfunctional) vehicular screen-wiper.
No two leaps may cover the same distance; no cell may be visited more than once.
It is easily seen that the rook can continue to move indefinitely; can it in the process also execute every size of leap, and/or eventually visit every cell?

(#1) **Formal Definition:**
Given some `leap' map U from |N to |N (natural numbers), denote by U' the `cell' alternating partial sum
U'(n) = -\sum_{0 <= k <= n} (-1)^k U(k)
= - U(0) + U(1) - U(2) + ... +/- U(n) .
U has the "rook-wiper" property when both U, U' are injective; Angelini's rook-wiper sequence S is defined as the lexicographically earliest such U --- see #2.

(#2) **Examples:**

Original, S earliest ---
n 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
S(n) 0 2 1 5 3 6 4 7 8 12 9 11 10 15 13 17 14 16 18 22 19 21 20 25 23
S'(n) 0 2 1 6 3 9 5 12 4 16 7 18 8 23 10 27 13 29 11 33 14 35 15 40 17

Variant, T' earliest ---
n 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
T(n) 0 2 1 5 3 6 4 7 8 12 9 11 10 15 13 17 16 18 14 19 21 23 22 25 20
T'(n) 0 2 1 6 3 9 5 12 4 16 7 18 8 23 10 27 11 29 15 34 13 36 14 39 19

Sparse rook-wiper sequence ---
n 0 1 2 3 4 5 6 7 8 9 10 ... (2 k - 1) (2 k)
X(n) 0 4 1 8 3 16 7 32 15 64 31 ... (2 2^k) (2^k - 1)
X'(n) 0 4 3 11 8 24 17 49 34 98 67 ... (3 2^k + k - 3) (2 2^k + k - 2)

(#3) **Existence & construction:**
The sparse example X in #2 illustrates that infinite rook-wipers U exist;
whence it follows that the unique earliest one S also exists.
Furthermore X illustrates how for odd n , any initial segment U(0..n)
satisfying the constraints can always be continued indefinitely, via subsequent
escape to some arbitrarily large doublet U(n+1), U(n+2) of `fat' and `thin'
entries, satisfying
U(n+1) notin U(0..n) , U(n+2) notin U(0..n) ,

U'(n+1) notin U'(0..n) , U'(n+2) notin U'(0..n) ,
where
U'(n+1) - U'(n) = U(n+1) , U'(n+1) - U'(n+2) = U(n+2) .
It follows that any given odd-length segment of S can be computed finally
and directly, by selecting the earliest available doublet at each double step;
it is never necessary to backtrack from further trials at greater length.

(#4) **Permutations:**
Now a surprise emerges from hiding in plain sight: both S, S' appear
not merely injective but surjective, so yielding permutations of |N !
For example if n = 1000, 10000, segment S(0..n) yields an exact
permutation of interval (0..n) ; if n = 10000 the only deviation is at
S(100000) = 100003 . However S' is less strict: only about 70% of
S'(0..n) seems always to fall within (0..n) .

(#5) **Asymptotics:**
More specifically, I conjecture that as n -> oo , the ratios
n : min(k | k notin S(0..n)) : max(k | k in S(0..n))
: min(k | k notin S'(0..n)) : max(k | k in S'(0..n))
approach exact limits
1 : 1 : 1 : 1/sqrt2 : 1 + 1/sqrt2 ,
or approximately
1.0000 : 1.0000 : 1.0000 : 0.7071 : 1.7071 .
Any actual proof seems a distant prospect; perhaps some probabilistic argument
might assist in elucidating this remarkable behaviour?

(#6) **Variations:**

Don't go away, there's more to come! Contemplation of #3 suggests that
lexicographical ordering of U in the definition of S may have been perhaps
somewhat arbitrary: why not amend that, to consider instead the rook-wiper T
for which T' is lexicographically earliest? Comparing the resulting sequences
--- see #2 --- it becomes apparent that S,T seem practically indistinguishable:
equal for long stretches, sharing identical surjective and asymptotic behaviour,
and effectively highly localised deviations from the identity permutation.

Worse still, variant Q combining earliest doublet Q(2k-1),Q(2k) for k odd,
Q'(2k-1),Q'(2k) for k even, behaves similarly: eg. Q(n) = S(n) for n < 42 .
At which juncture, the well-known principle of engineer's induction makes it
inevitable to speculate that _every_ mixed strategy selecting earliest doublets
for either of U, U' delivers such a typical pair of rook-wiper permutations;
and this conjecture is supported by a computer tests involving random selection.

(#7) **Permutation Questions:**
More generally,
Let U, U' denote injections from |N to |N ,
and suppose that for all n in |N ,
U'(n) = - U(0) + U(1) - U(2) + ... +/- U(n) .
Is it true that U' is surjective (only) if U is?

Proofs and constructions of simple counter/examples are solicited!

(#8) **Computer Program:**
The final Test section of the MAGMA program source attached at
https://www.dropbox.com/s/4m4y3er1n6dhco9/rook_wiper_prog.txt
may quickly be edited to produce any of the permutation sequences above.
An adventurous user can customise dynamic doublet selection between between
earliest leap & cell, by modifying a Boolean function of the step number.
The complete program may be executed via copy-paste into a free online
calculator at
http://magma.maths.usyd.edu.au/calc/

(#9) **OEIS:**
Surely this topic deserves at least a quartet of entries in OEIS, incorporating
sequences S,S',T,T' , a selective summary of the material above, and perhaps
Bickford's program and diagram.

Fred Lunnon, Maynooth 08/07/21

MFH *10 juillet 2021 à 21:08*

A rook as I know it moves in 4 directions, 2D, this here moves in only 2 directions, 1D, rather like a Turing machine going left and right on a 1D tape.

I don't see why this is mentioning a rook... Why not a Queen, then?

RÉPONDRE

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Posts les plus consultés de ce blog

A square for three (chess)

juin 22, 2024

(English translation after the French text) Voici cinq problèmes d'échecs disjoints : a) combien faut-il de coups au minimum pour que trois pions soient capturés sur la même case ? b) trois tours c) trois c …

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Le tripalin se présente

avril 11, 2024

Un tripalin est constitué de trois images. Chaque image illustre un substantif. Accolés, ces trois substantifs forment une chaîne palindromique. Laquelle nous vous invitons à trouver. Exem …

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Some strings au cinéma Galeries

juillet 19, 2024

Lettre ouverte au cinéma Galeries   Bonsoir à tous,   Je viens de voir pour la seconde fois chez vous le beau film de Léos Carax (la première fois c'était le 26 juin en présence du réalisateur, au BRIFF). Apparut à l'écran aujourd'hui, avant la projection propre …

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ÉRIC ANGELINI

CONSULTER LE PROFIL

Archiver