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1 Early computations on n! as a sum of three tetrahedral numbers (A000292)

Conjecture 1. *$n! = \binom{x}{3} + \binom{y}{3} + \binom{z}{3}$ is always soluble for any $n \geq 0$ where $x, y, z \geq 2$.*

$$1! = \binom{2}{3} + \binom{2}{3} + \binom{3}{3}$$

$$2! = \binom{2}{3} + \binom{3}{3} + \binom{3}{3}$$

$$3! = \binom{4}{3} + \binom{4}{3} + \binom{5}{3}$$

$$4! = \binom{2}{3} + \binom{4}{3} + \binom{6}{3}$$

$$5! = \binom{2}{3} + \binom{2}{3} + \binom{10}{3}$$

$$6! = \binom{6}{3} + \binom{6}{3} + \binom{17}{3}$$

$$7! = \binom{8}{3} + \binom{21}{3} + \binom{29}{3}$$

$$8! = \binom{20}{3} + \binom{32}{3} + \binom{60}{3}$$

$$9! = \binom{17}{3} + \binom{84}{3} + \binom{118}{3}$$

$$10! = \binom{10}{3} + \binom{202}{3} + \binom{240}{3}$$

$$11! = \binom{71}{3} + \binom{95}{3} + \binom{621}{3}$$

$$12! = \binom{336}{3} + \binom{985}{3} + \binom{1236}{3}$$

$$13! = \binom{390}{3} + \binom{596}{3} + \binom{3336}{3}$$

$$14! = \binom{785}{3} + \binom{2704}{3} + \binom{7953}{3}$$

$$15! = \binom{550}{3} + \binom{6601}{3} + \binom{19626}{3}$$

$$16! = \binom{322}{3} + \binom{1512}{3} + \binom{50072}{3}$$

$$17! = \binom{3185}{3} + \binom{46852}{3} + \binom{126646}{3}$$

$$18! = \binom{58178}{3} + \binom{73785}{3} + \binom{335654}{3}$$

$$19! = \binom{1240}{3} + \binom{685510}{3} + \binom{741526}{3}$$

$$20! = \binom{21128}{3} + \binom{794310}{3} + \binom{2415654}{3}$$

$$21! = \binom{938012}{3} + \binom{2805276}{3} + \binom{6570392}{3}$$

$$22! = \binom{1392457}{3} + \binom{12388354}{3} + \binom{16915438}{3}$$

$$23! = \binom{1046882}{3} + \binom{1652244}{3} + \binom{53729148}{3}$$

$$24! = \binom{11393632}{3} + \binom{118661020}{3} + \binom{127041926}{3}$$

$$25! = \binom{8010871}{3} + \binom{54645963}{3} + \binom{452908834}{3}$$

$$26! = \binom{16338401}{3} + \binom{166999562}{3} + \binom{1341665521}{3}$$