Call a nonnegative integer *legal* if it is in A377912. That is, a number is legal if it does not contain an even digit d immediately followed by a digit $q \leq d$. Let S denote the sequence A342042, and $a(n)$ the n-th term of this sequence. We know that every legal number appears in S.

Theorem. *Let $d \in \{1, 2, ..., 8\}$. For every positive integer k, the k-digit number $d99...9$ appears in S before the k-digit number $(d+1)99...9$.*

Proof. Suppose not, and that k and d are minimal such that this fails. Suppose that $a(n_0) = (d+1)99...9$ and $a(n_1) = d99...9$, where $n_1 > n_0$.

Case 1: If d is odd, then $d+1$ is even. Then, $a(n_0 - 1)$ ends with a digit strictly less than $d+1$, or an odd digit. However, this means that $d99...9$ would have been a possible candidate for $a(n_0)$. Since $d99...9 < (d+1)99...9$, we get a contradiction.

Case 2: When d is even.
Claim: Then,

$$\begin{aligned} &|\{n \leq n_0 : a(n) \text{ has } k \text{ digits and ends with } d\}| \\ &\geq |\{n \leq n_0 : a(n) \text{ has } k \text{ digits and starts with } d+1\}| \end{aligned} \tag{1}$$

Suppose not. Then some k-digit number $(d+1)d_2...d_k$ appears directly after a k-digit number of the form $d'_1...d'_{k-1}d'_k$, where d'_k is either odd or less than d. In either case, $d99...9$ would have been possible in the place of $(d+1)d_2...d_k$, a contradiction.

Now, notice that when d is even:

$$\begin{aligned} &|\{n \in \mathbb{N} : a(n) \text{ has } k \text{ digits and ends with } d\}| \\ &< |\{n \in \mathbb{N} : a(n) \text{ has } k \text{ digits and starts with } d+1\}| \end{aligned} \tag{2}$$

Since every k-digit legal number has appeared when we reach n_0, we get that (1) contradicts (2). □