Call a nonnegative integer *legal* if it is not in A347298. That is, a number is legal if it does not contain an even digit d immediately followed by a digit $q \leq d$. Trivially, A342042 contains only legal numbers.

Theorem. *A342042 contains every legal number.*

Proof. Suppose there's a legal number that does not appear in the sequence. Let N be the smallest such counterexample. Then for some index n, $\{a(1), ..., a(n)\}$ contains every legal number less than N. If for some $n' > n$, $a(n')$ is odd, then the next term will be the smallest legal number not yet in the sequence. Thus, $a(n'+1) = N$, a contradiction. Hence, it is sufficient to prove that the sequence contains infinitely many odd numbers.

Suppose that for some n_0, $a(n)$ is even for every $n > n_0$. Fix an even digit d such that the sequence contains infinitely many terms ending with d. At least one such d exists. Let $n > n_0$ be such that $a(n)$ ends with d. By our assumption, the next term, $a(n+1)$, is even. Let $M = a(n+1)$ be this term. M starts with a digit larger than d and is legal. The goal is to show that $M+1$ appears in the sequence at an index larger than n_0. As $M+1$ is odd, this yields a contradiction.

Claim: $M+1$ is legal and does not appear in the sequence before M. (1)

Proof of the claim: M is legal and ends with an even digit q (which may or may not be equal to d). Then $M+1$ has the same digits as M except that it ends with the odd digit $q+1 > q$. Clearly it is legal. Now, suppose that it appears in the sequence before M. That is, there's some index $k < n+1$ such that $a(k) = M+1$. However, then it would also be possible to extend the sequence at position k with the value M. This is a smaller value, so it is a contradiction. □

Now, let $n' > n$ be the next index such that $a(n')$ ends with d. Such an index exists by our assumption on d.

Case 1: $M+1$ does not appear before index n': Then, since the last time a term ended with d (at index n), the smallest possible value that could extend the sequence was M, we now get that the smallest possible value that can extend the sequence is $M+1$. Thus, $a(n'+1) = M+1$.

index	n	$<$	$n+1$	$\leq$	n'	$<$	$n'+1$
value of the sequence	ends with d		M		ends with d		$M+1$

Case 2: $M+1$ appears before index n': Then by (1), there's n'' such that $n+1 < n'' < n'$, and $a(n'') = M+1$.

index	n	$<$	$n+1$	$<$	n''	$<$	n'
value of the sequence	ends with d		M		$M+1$		ends with d

In both cases, we get that $M+1$ is a term of the sequence appearing at an index larger than n_0.

□