

To show that the maximum number of distinct resistances is divisible by two, first we note that it suffices to consider pairs of resistances x and $1/x$ that are inverses. If we have a circuit of resistances w and y we can scale all the resistances by dividing by \sqrt{wy} so that $x = w/\sqrt{wy}$ and $1/x = y/\sqrt{wy}$.

Now we construct for each circuit having total resistance r a unique dual circuit having resistance $1/r$. If the network has one resistor, then the dual network has one resistor with the inverse resistance. If the network has more than one resistor then it consists either (exclusively) of two or more sub networks in series, or parallel. If it consists of subnetworks connected in series then the dual of the network consists of the duals of the subnetworks connected in parallel, and if it consists of subnetworks in parallel then the dual of the network consists of the duals of the subnetworks connected in series. It is easy to see by induction that the resistance of the dual is the inverse of the resistance of the original network, and that taking the dual is an involution. Note that the dual of a circuit is the planar dual of the underlying series-parallel graph with the resistances swapped.

It remains to show that there are no self dual circuits. We show that there is no series parallel arrangement of resistors that has resistance one independent of the values x and $1/x$ of the resistors. Note that the power dissipated by the circuit is the sum of the powers dissipated across the resistors. Now if we have a circuit of n resistors of resistances x and $1/x$ with $x > n$ then it cannot have total resistance one. Assume the contrary, then if we apply one Volt at the terminals, one Amp of current will flow, dissipating one Watt, but there is at most one Volt across each resistor of resistance x , so it dissipates at most $1/x$ Watts, and at most one amp of current flowing through each resistor of resistance $1/x$, so it too dissipates at most $1/x$ Watts. So the circuit dissipates at most $n/x < 1$ Watts, contradicting our assumption that it drew one Watt.

Thus, we can pair each circuit with a circuit having the inverse resistance, and so the total number of distinct resistances is divisible by 2.