

Cinquante signes

The box ■ operation



décembre 06, 2019



Hello SeqFans,

Let's define $(a \blacksquare b) = c$ with an example:

$$\begin{array}{r}
 12951 \\
 \blacksquare \quad 2019 \\
 \hline
 = 10948
 \end{array}$$

We align a and b on the right and make the absolute differences of the vertically disposed digits. For example, the 8 above comes from $1 - 9$ and the 4 from $5 - 1$.

The result 10948 starts with 1 as this 1 comes from $1 - 0$ (the 0 being "invisible" though).

The *box* algebra is fun to explore – note for instance that one always has $(a \blacksquare b) = (b \blacksquare a)$, and $(a \blacksquare b) \blacksquare c = a \blacksquare (b \blacksquare c)$, etc. [NO for the associativity! Read **Maximilian** below!]

Here is a sequence **S** of numbers such as $(n \blacksquare k)$ is always a square, k being the smallest possible integer:

S = 2, 1, 2, 3, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, 5, 6, 7, 8, 9, 21, 1, 2, 3, 4, 6, 7, 8, 9, 19, 10, 11, 1, 2, 3, 9, 17, 18, 19, 29, 20, 10, 11, 12, 13, 19, 27, 28, 29, 39, 30, 20, 21, 22, 10, 4, 5, 6, 7, 8, 1, ...

Example:

For $n = 1$ the smallest k producing a square is 2 (as $1 \blacksquare 2 = 1$, this 1 being the square of 1);

For $n = 2$ the smallest k producing a square is 1 (as $2 \blacksquare 1 = 1$, this 1 being the square of 1);

For $n = 3$ the smallest k producing a square is 2 (as $3 \blacksquare 2 = 1$, this 1 being the square of 1);

For $n = 4$ the smallest k producing a square is 3 (as $4 \blacksquare 3 = 1$, this 1 being the square of 1);

For $n = 5$ the smallest k producing a square is 3 (as $5 \blacksquare 1 = 4$, this 4 being the square of 2);

For $n = 6$ the smallest k producing a square is 3 (as $6 \blacksquare 2 = 4$, this 4 being the square of 2);

...

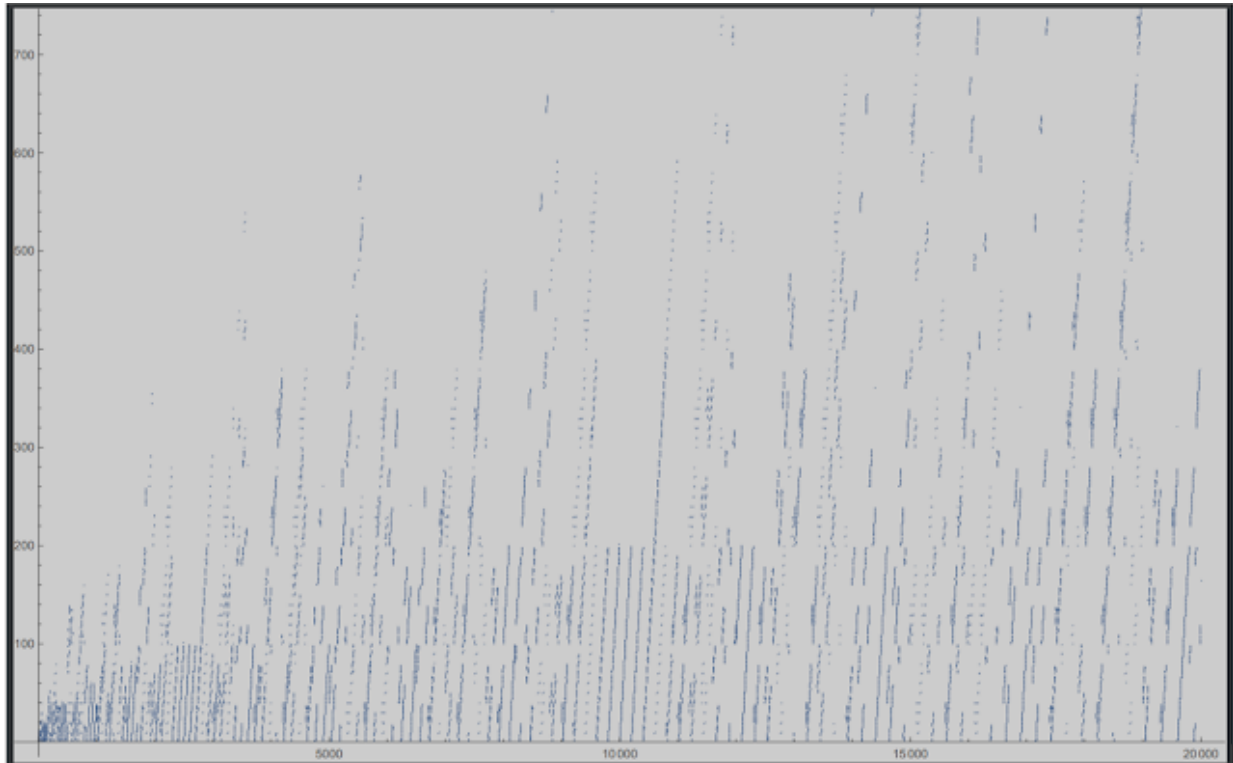
For $n = 16$ the smallest k producing a square is 12 (as $16 \blacksquare 12 = 4$, this 4 being the square of 2);

For $n = 17$ the smallest k producing a square is 1 (as $17 \blacksquare 1 = 16$, this 16 being the square of 4);

etc.

My friend **Jean-Marc Falcoz** has computed the first 20000 terms of **S**. He writes me that the highest k is 2175 so far (with $16575 \blacksquare 2175 = 14400$, square of 120) – the missing k s so far being 565, 678, 680, 681, etc.

The graph of **S** is emblematic of the rainfalls that affect Brussels for at least another week!



Have fun until the ■ [Boxing Day!](#)

Best,
É.



MFH 6 décembre 2019 à 05:38

Very funny! Yes, the ■ operation has the nice properties of being commutative, associative, and having a neutral element $e = 0$ ($x \blacksquare 0 = 0 \blacksquare x = x$) and every element x has a symmetric x' such that $x \blacksquare x' = e$, namely itself: $x' = x$. This makes it an abelian group. So it is natural to think of it as a fancy law of addition. One might want to find a "compatible" multiplication, i.e., distributive over ■. Then it might be natural to consider squares for that multiplication, rather than the usual one.

(The property $x+x=0$ holds in [https://en.wikipedia.org/wiki/Boolean_ring Boolean rings] characterized by $x^2 = x$ for all x . But in that case, "squares" are obviously not very interesting...)



MFH 6 décembre 2019 à 06:25

Oops, actually ■ is **not** associative as you wrote $[(a \blacksquare b) \blacksquare c =?= a \blacksquare (b \blacksquare c)]$, consider e.g., $a=2, b=c=1$. [Else you could find the $a(n)=k$ you are looking for as $\min \{ x^2 \blacksquare n, y^2 \blacksquare n \} \setminus \{0\}$, where x^2 is the largest square $< n$ and y^2 is the smallest square $> n$, I think.]



Éric ANGELINI 6 décembre 2019 à 08:06

Geeee, glad there is someone reading my posts! Thank you Maximilian!

RÉPONDRE

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A square for three (chess)

juin 22, 2024



(English translation after the French text) Voici cinq problèmes d'échecs disjoints : a) combien faut-il de coups au minimum pour que trois pions soient capturés sur la même case ? b) trois tours c) trois c ...

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Le tripalin se présente

avril 11, 2024



Un tripalin est constitué de trois images. Chaque image illustre un substantif. Accolés, ces trois substantifs forment une chaîne palindromique. Laquelle nous vous invitons à trouver. Exer ...

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Some strings au cinéma Galeries

juillet 19, 2024

Lettre ouverte au cinéma Galeries Bonsoir à tous, Je viens de voir pour la seconde fois chez vous le beau film de Léos Carax (la première fois c'était le 26 juin en présence du réalisateur, au BRIFF). Apparus à l'écran aujourd'hui, avant la projection propre ...

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