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Why these terms are Hihgly Brazilian (HB) and not highly composite (HC) ?
A309493

Given:

The number of divisors of the integer n is τ(n).
The number of ways for a number n to be Brazilian is called β(n) and
β(n) = β'(n) + β''(n) where

- β'(n) is the number of representations with only two digits as aa_b , and
- β''(n) is the number of representations with at least three digits.

If the relation β = f(τ) between the number of Brazilian representations and the number of divisors of n was the same for all the integers, the highly composites would be the same than the highly Brazilian numbers.

For non-oblong and nonsquares numbers, β'(n) = τ(n)/2 – 1, and,
for oblong numbers, β'(n) = τ(n)/2 – 2.

Why the six known terms of A309493 are HB but not HC ?
There are two types of explanations:

1) For 7, 15 and 40, it is because these three terms have a Brazilian representation with 3 digits or 4 digits and they belong to A326380.

n	τ(n)	β(n)	β'(n)	β''(n)	Integer	Relation β = f(τ)
2	2	0	0	0	prime	β(n) = τ(n)/2 - 1
7	2	1	0	1	prime	β(n) = τ(n)/2
6	4	0	0	0	oblong	β(n) = τ(n)/2 - 2
15	4	2	1	1	non-oblong	β(n) = τ(n)/2
24	8	3	3	0	non-oblong	β(n) = τ(n)/2 - 1
40	8	4	3	1	non-oblong	β(n) = τ(n)/2

Exactly,

7 = 111_2 , 7 is the smallest Brazilian number with β(7) = 1, then τ(7) = τ(2) = 2 with 2 < 7, hcnce 7 cannot be HC.

15 = 1111_2 = 33_4 **,** 15 is the smallest 2-Brazilian number with β(15) = 2, then τ(15) = τ(6) = 4 with 6 < 15, hence 15 cannot be HC.

$40 = 1111_3 = 55_7 = 44_9 = 22_{19}$, 40 is the smallest 4-Brazilian number with β(40) = 4, then τ(40) = τ(24) = 8 with 24 < 40, hence 40 cannot be HC.

2) For 336, 1440 and 5405400, called r. It is because each of these HB terms r is non-oblong and the greatest HC m less than r is oblong with the same number of divisors.

Exactly:

r is non-oblong and belongs to A326386, this term has a number of divisors equal to τ(r) = 2*q, hence β(r) = τ(r)/2 – 1 = q-1. The greatest HC number m < r is oblong and has the same number of divisors as r, so τ(m) = τ(r) = 2*q. As m is oblong and belongs to A326379, this term satisfies the relation β(m) = τ(m)/2 – 2 = q-2. We have r that cannot be HC as τ(r) = τ(m) = 2*q with m < r, and β(r) = β(m) + 1, hence r is HB.

In this array, the respective triplets (r, m, 2*q) are (336, 240, 20), (1440, 1260, 36) and (5405400, 4324320, 384).

n	τ(n)	β(n)	Integer	Relation β = f(τ)	HC	HB
180	18	8	non-oblong	β(n) = τ(n)/2 - 1	Yes	Yes
240	20	8	oblong	β(n) = τ(n)/2 - 2	Yes	No
336	20	9	non-oblong	β(n) = τ(n)/2 - 1	No	Yes
1260	36	16	oblong	β(n) = τ(n)/2 - 2	Yes	Yes
1440	36	17	non-oblong	β(n) = τ(n)/2 - 1	No	Yes
4324320	384	190	oblong	β(n) = τ(n)/2 - 2	Yes	Yes
5405400	384	191	non-oblong	β(n) = τ(n)/2 - 1	No	Yes

The first row begins with 180 because 240 is HC but not HB (A309039), and 180 is HC and HB. In the two other cases, 1260 and 4324320 both are HB and HC.

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