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Material from Bill Gosper's Computers
& Math talk, M.I.T., 1989

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$$
{}_4F_3\left[\begin{matrix} a-\frac{1}{5}, & a, & a+\frac{1}{5}, & \frac{5a+2}{3} \\ & \frac{5a+1}{4}, & \frac{5a+3}{4}, & \frac{5a-1}{3} \end{matrix}\middle|\, \frac{125}{128}\right] = \frac{\pi}{(a-\frac{1}{5})!}\,\frac{\sqrt{1+\sqrt{5}}}{(a+\frac{1}{5})!}\,\frac{\frac{5a+1}{2}!}{\frac{a-1}{2}!}\,\frac{2^{5a+\frac{3}{2}}}{5^{\frac{5}{2}a+\frac{5}{4}}}
$$

$$
{}_4F_3\left[\begin{matrix} a-\frac{1}{5}, & a, & a+\frac{1}{5}, & \frac{4-5a}{3} \\ & \frac{1}{2}, & 5a, & \frac{1-5a}{3} \end{matrix}\middle|\, \frac{125}{128}\right] = \sqrt{\frac{1-1/\sqrt{5}}{\pi}}\,\frac{(a-\frac{3}{5})!\,(a-\frac{2}{5})!}{(2a-\frac{1}{2})!}\,2^{a-\frac{1}{2}},
$$

$$
{}_4F_3\left[\begin{matrix} a-\frac{1}{5}, & a, & a+\frac{1}{5}, & \frac{8-5a}{3} \\ & \frac{3}{2}, & 5a-2, & \frac{5-5a}{3} \end{matrix}\middle|\, \frac{125}{128}\right] = \sqrt{\frac{1-1/\sqrt{5}}{\pi}}\,\frac{(a-\frac{7}{5})!}{1-a}\,\frac{(a-\frac{3}{5})!}{(2a-\frac{3}{2})!}\,\frac{2^{a+\frac{3}{2}}}{5}.
$$

$$
N_{k,n} := \begin{pmatrix} \frac{n-\frac{k}{2}}{n+1}\,\frac{n-\frac{k-1}{2}}{n+2k+\frac{3}{2}}\,\frac{1}{5} & 1 \\ 0 & 1 \end{pmatrix}
$$

$$
K_{k,n} := \begin{pmatrix} \frac{k+\frac{4}{5}}{k-2n+1}\,\frac{k+1}{k+\frac{n}{2}+\frac{3}{4}}\,\frac{k+\frac{6}{5}}{k+\frac{n}{2}+\frac{5}{4}}\,\frac{125}{128} & -\frac{15}{16}\,\frac{n}{k-2n+1}\,\frac{k+\frac{4}{3}n+\frac{2}{3}}{k+\frac{n}{2}+\frac{3}{4}} \\ 0 & 1 \end{pmatrix},
$$

$$
M_{k,n} := \begin{pmatrix} -\frac{15/16}{k-2n+1}\,\frac{n}{k+\frac{n}{2}+\frac{3}{4}} & 0 \\ 0 & 1 \end{pmatrix}
$$

$$
N'_{k,n} := M_{k,n}^{-1} N_{k,n} M_{k,n+1} = \begin{pmatrix} \frac{n-\frac{k}{2}}{n}\,\frac{n-\frac{k+1}{2}}{n+2k+\frac{5}{2}}\,\frac{1}{5} & 16\,\frac{n+2k+\frac{3}{2}}{15}\,\frac{n-\frac{k+1}{2}}{n} \\ 0 & 1 \end{pmatrix},
$$

$$
K'_{k,n} := M_{k,n}^{-1} k_{k,n} M_{k+1,n} = \begin{pmatrix} \frac{k+\frac{4}{5}}{k-2n+2}\,\frac{k+1}{k+\frac{n}{2}+\frac{5}{4}}\,\frac{k+\frac{6}{5}}{k+\frac{n}{2}+\frac{7}{4}}\,\frac{125}{128} & k+\frac{4n+2}{3} \\ 0 & 1 \end{pmatrix}.
$$

$$N''_{k,n} := N'_{k+2n-1,n}\,K'_{k+2n-1,n+1}\,K'_{k+2n,n+1} = \mathrm{Mess}(k,n)$$

$$K''_{k,n} := K'_{k+2n-1,n} = \begin{pmatrix} \frac{k+2n-\frac{1}{5}}{k+1}\frac{k+2n}{k+\frac{5}{2}n+\frac{1}{4}}\frac{k+2n+\frac{1}{5}}{k+\frac{5}{2}n+\frac{3}{4}}\frac{125}{128} & k+\frac{10n-1}{3} \\ 0 & 1 \end{pmatrix}.$$

$$N''_{0,n} = \begin{pmatrix} \frac{n-\frac{1}{10}}{n+\frac{3}{10}}\frac{n+\frac{2}{5}}{n+\frac{7}{10}}\frac{n+\frac{3}{5}}{n+\frac{9}{10}}\frac{1}{64} & 0 \\ 0 & 1 \end{pmatrix}$$

$$\prod_{n=x}^{y}(n+a) = \frac{(y+a)!}{(x+a-1)!}$$

$$\prod_{n=x}^{y}(-n-a) = \frac{(-x-a)!}{(-y-a-1)!} = \frac{(y+a)!}{(x+a-1)!}\frac{\sin\pi(y+a)}{\sin\pi(x+a-1)}$$

$$N'''_{k,n} := N'_{k-n-1,2n+3/2}\,N'_{k-n-1,2n+5/2}\,K'^{-1}_{k-n-2,2n+7/2} = \mathrm{Yuck}(k,n),$$

$$K'''_{k,n} := K'_{k-n-1,2n+3/2} = \begin{pmatrix} \frac{k-n-\frac{1}{5}}{k+1}\frac{k-n}{k+\frac{3}{2}}\frac{k-n+\frac{1}{5}}{k-5n-2}\frac{125}{128} & k+\frac{5n+5}{3} \\ 0 & 1 \end{pmatrix}.$$

$$N'''_{0,n} = \begin{pmatrix} \frac{n+\frac{3}{5}}{n+\frac{3}{4}}\frac{n+\frac{7}{5}}{n+\frac{5}{4}}\frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$$

$$\prod_{n=x}^{y}(1-q^{n+a}) = (q^{x+a};q)_{y-x+1} = \frac{(y+a)!_q}{(x+a-1)!_q}(1-q)^{y-x+1}$$

$$\prod_{n=x}^{y}-(1-q^{n+a}) = \prod_{n=x}^{y}q^{n+a}(1-q^{-n-a}) = q^{(y-x+1)(\frac{x+y}{2}+a)}(q^{-y-a};q)_{y-x+1}$$

$$= q^{(y-x+1)(\frac{x+y}{2}+a)} \frac{(-x-a)!_q}{(-y-a-1)!_q}(1-q)^{y-x+1}$$

$$= \frac{(y+a)!_q}{(x+a-1)!_q} \frac{\sin_{\sqrt{q}} \pi(y+a)}{\sin_{\sqrt{q}} \pi(x+a-1)}(1-q)^{y-x+1}.$$

$$\sum_{k=1}^{n-1} \frac{(x+a)^{-k}}{k} \frac{x^{-(n-k)}}{n-k} = [z^n] \ln\left(1-\frac{z}{x+a}\right) \ln\left(1-\frac{z}{x}\right)$$

$$= \sum_{k=1}^{n-1} \frac{a^{k-n}}{(n-k)k} \left(\frac{(-)^{n-k}}{\binom{n}{k}} - \frac{k}{n}\right) \left(\frac{1}{(x+a)^k} + \frac{(-)^{n-k}}{x^k}\right).$$

$$x = \frac{b}{a - \ln x} = \frac{1}{1 - \sum_{n\geq 0} b^{-n-1} \sum_k \frac{(b-a)^{n-k+1}}{(n-k+1)!} \left[{n \atop k}\right]}$$

$$= \sum_{n\geq 1} b^{1-n} \sum_k \frac{(a-b)^{n-k}}{(n-k)!} \sum_{j=0}^{k} (-)^{n-j} \left[{n \atop j}\right]$$

$$=: f(a,b)$$

$$= cf(a - \ln c, b/c)$$

where $\left[{n \atop k}\right]$ are the first kind of Stirling numbers, $\sum_{k\geq 0} \left[{n \atop k}\right] x^k = x(x+1)\dots(x+n-1)$.

$$2\sum_{k\geq 0} J'_{2k}(2kz) = 1\frac{z}{1!} + 4\frac{u^3}{3!} + 66\frac{u^5}{5!} + 2416\frac{u^7}{7!} + \cdots$$

A25585

$$= \left\langle {1 \atop 0} \right\rangle \frac{z}{1!} + \left\langle {3 \atop 1} \right\rangle \frac{u^3}{3!} + \left\langle {5 \atop 2} \right\rangle \frac{u^5}{5!} + \left\langle {7 \atop 3} \right\rangle \frac{u^7}{7!} + \cdots$$

$$2\sum_{k\geq 0} (-)^k \frac{J_{2k+1}\left((2k+1)z\right)}{2k+1} = z\cos(z\cos(z\dots))$$

A309204 / A309205 (and A309206)

$$= z - \frac{z^3}{2} + \frac{13\,z^5}{24} - \frac{541\,z^7}{720} + \frac{9509\,z^9}{8064} - \frac{7231801\,z^{11}}{3628800} + O(z^{13}).$$

$$\sum_{n\geq 1}\left(n\pi+\arctan\frac{n\pi+\arctan\frac{n\pi+\cdots}{3}}{3}\right)^{-2} = (3\tanh(3\tanh 3\ldots))^{-2}$$

A309208

$$\approx .1122528288730582219511299311388265401208$$

$$\approx [2,1,64,2,1,1,1,3,4,1,2,3,1,272213,1,2,1,\ldots]^{-2}$$

A309206 A309207

$$\sin\left(\sum_{j=1}^{n}\alpha_j\right) = \lim_{\epsilon\to 0}\epsilon\sum_{j=1}^{n}\prod_{k=1}^{n}\cos(\alpha_k)+\sin(\alpha_k)\cot(\beta_k-\beta_j+\epsilon)$$

$$= \sum_{j=1}^{n}\sin\alpha_j\prod_{\substack{k=1\\k\neq j}}^{n}\cos(\alpha_k)+\sin(\alpha_k)\cot(\beta_k-\beta_j).$$

$$\tan(\alpha-\beta)\,\tan(\beta-\gamma)\,\tan(\gamma-\alpha) = \tan(\alpha-\beta)+\tan(\beta-\gamma)+\tan(\gamma-\alpha).$$

$$\sum_{j=1}^{n}\prod_{\substack{k=1\\k\neq j}}^{n}\cot(\beta_j-\beta_k) = \sin n\frac{\pi}{2}$$

$$\sum_{j=1}^{n}\alpha_j = \sum_{j=1}^{n}\alpha_j\prod_{\substack{k=1\\k\neq j}}^{n}1+\frac{\alpha_k}{\beta_j-\beta_k}.$$

I.e., you can multiply the terms of any sum by this product of the other terms mixed up with these arbitrary βs, and get the same answer!

Choose $\alpha_k := a/k^2, \beta_j := 1/j$. Let $n\to\infty$:

$$\sum_{k\geq 1}\left(1-(-)^k\cos\sqrt{(k+a)k}\pi\right)\frac{\Gamma\left(\frac{k-\sqrt{(k+a)k}}{2}\right)\Gamma\left(\frac{k+\sqrt{(k+a)k}}{2}\right)}{kk!} = -\frac{a\pi^4}{12}, \qquad \Re a<4.$$

If, instead $\alpha_k := a/(k+b)k$:

$$\sum_{k\geq 1}\left(1-(-)^k\cos D_k\pi\right)\frac{\Gamma\left(\frac{k-D_k}{2}\right)\Gamma\left(\frac{k+D_k}{2}\right)}{(k+b)^2\Gamma(k)} = -\frac{\pi^2 a}{2b}H_b, \qquad \Re a<4.$$

$$D_k := \sqrt{k^2 + ak + ab},$$

$$H_b := \Psi_0(b+1) + \gamma \text{ ``=''} \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{b}.$$

$$\sum_{k\ge 1} \frac{H_k}{k^2} = 2\zeta(3),$$

$$\sum_{k\ge 1} \frac{H_k^{(2)}}{k^2} = \frac{\zeta(2)^2 + \zeta(4)}{2} = \frac{7\pi^4}{360},$$

$$\sum_{k\ge 1} \frac{H_k^2}{k^2} = \frac{17\pi^4}{360},$$

$$\sum_{k\ge 1} \frac{H_k^{(2)}}{k^3} = 3\zeta(2)\zeta(3) - \frac{9\zeta(5)}{2} \pm 10^{-12},$$

$$\sum_{k\ge 1} \frac{H_k^2}{k^3} = \frac{7\zeta(5)}{2} - \zeta(2)\zeta(3) \pm 10^{-12}.$$

$$H_k^{(2)} := \zeta(2) - \Psi_1(k+1) = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{k^2}.$$

$$\prod_{k=1}^{k} \begin{pmatrix} 1 & \frac{1}{k} & 0 \\ 0 & 1 & \frac{1}{k^2} \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & H_k & \sum_{k=1}^{k} \frac{H_{k-1}}{k^2} \\ 0 & 1 & H_k^{(2)} \\ 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & \zeta(1) & \zeta(3) \\ 0 & 1 & \zeta(2) \\ 0 & 0 & 1 \end{pmatrix}.$$

$$\prod_{n\ge 1} \frac{\sqrt{\pi}}{\sqrt[n]{4}} \frac{\frac{1}{n-1/2}!}{(\frac{1}{n} - \frac{1}{2})!} = 4^{-\gamma} = .4492432307139960293348606753923+.$$

$$\lim_{y\to 0} \frac{\displaystyle\sum_{n\ge 0} \frac{e^{-n((n-1)y+x)}}{\prod_{k=1}^{n} c\sinh(ky)\, c\sinh((k-1)y+x)}}{\displaystyle\sum_{n\ge 0} \frac{e^{-n(ny+x)}}{\prod_{k=1}^{n} c\sinh(ky)\, c\sinh(ky+x)}} = \frac{1 + \sqrt{1 + \frac{4}{(c\sinh x)^2}}}{2}.$$

$$c\sinh x+\cfrac{1}{c\sinh(x+y)+\cfrac{1}{c\sinh(x+2y)+\cfrac{1}{c\sinh(x+3y)+\ddots}}}$$

$$= c\sinh x\,\frac{\displaystyle\sum_{n\geq 0}\frac{e^{-n((n-1)y+x)}}{\prod_{k=1}^{n} c\sinh(ky)\,c\sinh\big((k-1)y+x\big)}}{\displaystyle\sum_{n\geq 0}\frac{e^{-n(ny+x)}}{\prod_{k=1}^{n} c\sinh(ky)\,c\sinh(ky+x)}}.$$

$$K_{k,n} := \begin{pmatrix} cz^k - y - \frac{1}{y} & -1 & \frac{1-z^{-n}}{c^k z^{(n-1)k}} \\ 1 & 0 & 0 \\ 0 & 0 & z^{-k} - \frac{yz^{-n}}{c} \end{pmatrix},$$

$$N_{k,n} := \begin{pmatrix} \frac{z}{cy(z^{n+1}-1)} & 0 & \frac{1}{c^k z^{kn}} - \frac{y}{c^{k+1} z^{kn+k+n}} \\ 0 & \frac{z}{cy(z^{n+1}-1)} & \frac{1}{c^{k+1} z^{kn+k+n}} \\ 0 & 0 & 1 - \frac{y}{cz^{n+k}} \end{pmatrix}.$$

$$\left(c-\frac{1}{y}\right)\frac{\displaystyle\sum_{n\geq 0}\prod_{k=1}^{n}\frac{y^2}{(1-z^{-k})(1-cyz^{k-1})}}{\displaystyle\sum_{n\geq 0}\prod_{k=1}^{n}\frac{y^2}{(1-z^{-k})(1-cyz^{k})}} = c-y-\frac{1}{y}-\cfrac{1}{cz-y-\frac{1}{y}-\cfrac{1}{cz^2-y-\frac{1}{y}-\cfrac{1}{cz^3-y-\frac{1}{y}-}}}.$$

$$(c+1)\,\frac{\displaystyle\sum_{n\geq 0}\frac{1}{\prod_{k=1}^{n}(z^{-k}-1)(1+cz^{k-1})}}{\displaystyle\sum_{n\geq 0}\frac{1}{\prod_{k=1}^{n}(z^{-k}-1)(1+cz^{k})}}=(c+i)\,\frac{\displaystyle\sum_{n\geq 0}\frac{1}{\prod_{k=1}^{n}\big((-z)^{-k}-1\big)\big(1-ic(-z)^{k-1}\big)}}{\displaystyle\sum_{n\geq 0}\frac{1}{\prod_{k=1}^{n}\big((-z)^{-k}-1\big)\big(1-ic(-z)^{k}\big)}}$$

$$=c+\cfrac{1}{cz+\cfrac{1}{cz^2+\cfrac{1}{cz^3+\ddots}}}$$

$$=c+\frac{1}{cz}-\frac{1}{c^3z^4}+\frac{1}{c^5z^7}+\frac{1}{c^5z^9}-\frac{1}{c^7z^{10}}-\frac{2}{c^7z^{12}}+\frac{1}{c^9z^{13}}$$

$$-\frac{1}{c^7z^{14}}+\frac{3}{c^9z^{15}}-\frac{c^4+1}{c^{11}z^{16}}+\frac{3}{c^9z^{17}}-\frac{4}{c^{11}z^{18}}+O(z^{-19}).$$

$$\sum_{n\geq 1}\left(-\frac{1}{2}\right)^n\frac{a+bz^{(-2)^{-n}}}{1+z^{(-2)^{-n}}}=\frac{a-b}{2}\left(\frac{2}{\ln z}+\frac{1+z}{1-z}\right)-\frac{a+b}{6}$$

$$\sum_{n\geq 1}(-)^n\left(az^{(-2)^{-n}/3}+bz^{5(-2)^{-n}/3}\right)\left(\sqrt[2^n]{z}-\frac{1}{\sqrt[2^n]{z}}\right)=(1-z^{-2/3})\big(a+b(z^{2/3}+1-z^{-2/3})\big)$$

$$\sum_{n\geq 0}(\sqrt{z}-c)\left(\sqrt{\sqrt{z}}-c\right)\ldots(\sqrt[2^n]{z}-c)=\frac{1+\frac{z}{c}}{1+c},\qquad |1-c|<1.$$

$\Longrightarrow$

$$\int_1^z \frac{dt}{t\prod_{k\ge1} 2t^{2^{-k}}-1} = \frac{2}{3}\int_{\sqrt{z}}^z \left(2+\frac{1}{t}\right)\frac{dt}{\prod_{k\ge1} 2t^{2^{-k}}-1},$$

$$\int_1^\infty \frac{dt}{t\prod_{k\ge1} 2t^{2^{-k}}-1} \stackrel{?}{=} \frac{4\ln 2}{3\prod_{k<\infty}\left(1+\alpha^{2^k}\right)\left(1-\alpha^{2^k}/2\right)} = .5718321+ \qquad \text{No!}$$

$$\int_1^z \frac{(\ln t)^{a-1}\,dt}{t\prod_{k\ge1} 1+2^a(t^{2^{-k}}-1)} = \frac{1}{2-2^{-a}}\int_{\sqrt{z}}^z \left(\frac{1}{1-2^{-a}}+\frac{1}{t}\right)\frac{(\ln t)^{a-1}\,dt}{\prod_{k\ge1} 1+2^a(t^{2^{-k}}-1)}.$$

$$\sum_{k\ge2}\arctan\frac{1}{k^2} = -\arctan\frac{\tanh\frac{\pi}{\sqrt{2}}}{\tan\frac{\pi}{\sqrt{2}}} = .639343615+,$$

$$\sum_{k\ge1}\operatorname{arctanh}\frac{2xy}{(k+\phi)^2-x^2-y^2} = \frac{1}{2}\ln\frac{(\phi-x-y)!\,(\phi+x+y)!}{(\phi-x+y)!\,(\phi+x-y)!},$$

$$\sum_{k}\operatorname{arctanh}\frac{2xy}{(k+\phi)^2-x^2-y^2} = \operatorname{arctanh}\frac{\sin 2\pi x\sin 2\pi y}{\cos 2\pi x\cos 2\pi y-\cos 2\pi\phi}.$$

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A309 2~~08~~

$$\sum_{k\ge1}\arctan\frac{(-)^k}{k} = \arg\frac{i-1}{2}! - \arg\frac{i}{2}! = -.50667090321662298+$$

A30920~~7~~9

$$= [-1, 2, 36, 1, 40, 1, 94, 1, 1, \ldots];$$

$$\sum_{k\ge1}\arctan\frac{6}{k(k^2+7)} = \arg\binom{3i}{i} = 1.22534477522635808682+.$$

$$E_k = \frac{\sum_{j=1}^{r}(-1)^{j+1}(1-2j)^k\sum_{i=0}^{r-j}\binom{r}{i}}{2^{r-1}}, \qquad r = \lfloor r\rfloor > k.$$

$$\sum_{k\geq 1}\arctan\frac{(-)^k}{k}=\arg\frac{i-1}{2}!-\arg\frac{i}{2}!=-.50667090321662298+$$

$$=-1_{+/}2_{+/}36_{+/}1_{+/}40_{+/}1_{+/}94_{+/}1_{+/}1_{+/}\cdots;$$

$$\sum_{k\geq 1}\arctan\frac{6}{k(k^2+7)}=\arg\binom{3i}{i}=1.22534477522635808682+.$$

$$(-)^{n+1}n\binom{x}{n}\prod_{k=0}^{n}\begin{pmatrix}\frac{k-n+1}{k+2} & (k+1)^p & 0\\ 0 & \frac{k-n}{k+1} & \frac{x-n}{x-k}\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}0 & 0 & \frac{B_{p+1}(x+1)-B_p}{p+1}\\ 0 & 0 & -n\\ 0 & 0 & (-)^{n+1}n\binom{x}{n}\end{pmatrix},$$

$$n\prod_{k=1}^{n}\begin{pmatrix}\frac{k-n}{k+1} & \frac{k-n}{k+1}k^p & k^{p-1}\\ 0 & \frac{k-n}{k+1} & \frac{1}{k}\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}0 & 0 & B_p(1)\\ 0 & 0 & H_n\\ 0 & 0 & n\end{pmatrix},$$

i.e.,

$$\sum_{k=1}^{n}\frac{(-)^{k-1}}{k}\binom{n}{k}\sum_{j=1}^{k}k^p=B_p(1),\qquad \forall n=\lfloor n\rfloor>p=\lfloor p\rfloor$$

Continuation of page 8 ?

$$\frac{1}{n}\left(\frac{1}{1}+\frac{1}{n+1}\left(\frac{1}{2}+\frac{2}{n+2}\left(\frac{1}{3}+\frac{3}{n+3}\left(\frac{1}{4}+\cdots\right)\right)\right)\right)=\frac{1}{n}+\frac{1}{2n^2}+\frac{1}{6n^3}-\frac{1}{30n^5}+\frac{1}{42n^7}-\cdots$$

$$=\sum_{k\geq 0}\frac{B_k(a+1)}{(n+a)^{k+1}}$$

$$=\frac{1}{n^2}+\sum_{k\geq 0}\frac{B_k(a)}{(n+a)^{k+1}}.$$

$$=\frac{1}{n^2}+\frac{1}{n+1}\left(\frac{1}{1}+\frac{1}{n+2}\left(\frac{1}{2}+\frac{2}{n+3}\left(\frac{1}{3}+\cdots\right)\right)\right)=\Psi_1(n).$$

$$\sum_k (k+\phi)\binom{a+b}{a+k+\phi}\binom{b+c}{b+k+\phi}\binom{c+d}{c+k+\phi}\binom{d+e}{d+k+\phi}\binom{e+f}{e+k+\phi}\binom{f+a}{f+k+\phi}$$

$$=\frac{(-b-f-1)!\,(-c-f-1)!\,(-d-f-1)!}{(a+c)!\,(c+e)!\,(e+b)!\,(b+d)!\,(d+a)!\,(a+e)!}\,\frac{\sin 2\phi\pi\sin(f-\phi)\pi\sin(f+\phi)\pi}{2\pi\sin(a+f)\pi\sin(e+f)\pi},$$

where

$$a+b+c+d+e+f=-1$$

and

$$\frac{\sin(a-\phi)\pi}{\sin(a+\phi)\pi}\frac{\sin(b-\phi)\pi}{\sin(b+\phi)\pi}\frac{\sin(c-\phi)\pi}{\sin(c+\phi)\pi}\frac{\sin(d-\phi)\pi}{\sin(d+\phi)\pi}\frac{\sin(e-\phi)\pi}{\sin(e+\phi)\pi}\frac{\sin(f-\phi)\pi}{\sin(f+\phi)\pi}=1.$$

$$A:=\pi a,\ B:=\pi b,\ C:=\pi c,\ D:=\pi d,\ E:=\pi e,$$

$$F:=\pi(f+1)=-A-B-C-D-E,$$

$$\varphi:=\pi\phi.$$

$$\frac{\sin(A-\varphi)\sin(B-\varphi)\sin(C-\varphi)\sin(D-\varphi)\sin(E-\varphi)}{\sin(A+\varphi)\sin(B+\varphi)\sin(C+\varphi)\sin(D+\varphi)\sin(E+\varphi)}=\frac{\sin(A+B+C+D+E-\varphi)}{\sin(A+B+C+D+E+\varphi)}$$

$\sin 2\varphi=0,\quad \bigcup$

$$\cos 2\varphi=\frac{\begin{matrix}\sin 2(E+F)\\ +\sin 2(D+E)+\sin 2(D+F)\\ +\sin 2(C+D)+\sin 2(C+E)+\sin 2(C+F)\\ +\sin 2(B+C)+\sin 2(B+D)+\sin 2(B+E)+\sin 2(B+F)\\ +\sin 2(A+B)+\sin 2(A+C)+\sin 2(A+D)+\sin 2(A+E)+\sin 2(A+F)\end{matrix}}{2(\sin 2A+\sin 2B+\sin 2C+\sin 2D+\sin 2E+\sin 2F)}.$$

$$\sum_{k\geq 1} \frac{k}{n}\binom{-n}{n-k}\mathrm{Fib}_{k+1} = 1$$

E.g., for $n = 36$,

$$\begin{array}{rrrr}
-\,3116285494907301262 & +\,6232570989814602524 & -\,6910024358272711494 & +\,7451987053039198670 \\
-\,7118315990962816640 & +\,6519730328086397968 & -\,5671008036856215984 & +\,4754773745187269524 \\
-\,3845772882136762115 & +\,3011206332934063005 & -\,2284279270870444848 & +\,1680041256669235005 \\
-\,1197916956272047830 & +\,827751016861286700 & -\,553857561325218150 & +\,358464245364285060 \\
-\,224093717913734760 & +\,135083021135715835 & -\,78354894431138325 & +\,43629048853904150 \\
-\,23254258186221020 & +\,11825370907939204 & -\,5715306457144704 & +\,2613440899230850 \\
-\,1124637287855575 & +\,452551815691118 & -\,168979435258491 & +\,57997008713371 \\
-\,18082344245680 & +\,5044456480310 & -\,1234269088962 & +\,257688946736 \\
-\,44100425171 & +\,5804075485 & -\,522562320 & +\,24157817
\end{array}$$

$$= 1\,.$$

$$\frac{k}{n}\binom{-n}{n-k} = \binom{-n}{n-k} + \binom{-n-1}{n-k-1},$$

$$\binom{-x}{y} = (-)^y\binom{y+x-1}{y}.$$

$$\begin{array}{rl}
\vdots & \\
n=-6 & 1 \quad \ldots \\
n=-5 & 1 \quad -5 \quad \ldots \\
n=-4 & 1 \quad -4 \quad 10 \quad \ldots \\
n=-3 & 1 \quad -3 \quad 6 \quad -10 \quad \ldots \\
n=-2 & 1 \quad -2 \quad 3 \quad -4 \quad 5 \quad \ldots \\
n=-1 & 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad \ldots \\
n=0 & 1 \\
n=1 & 1 \quad 1 \\
n=2 & 1 \quad 2 \quad 1 \\
n=3 & 1 \quad 3 \quad 3 \quad 1 \\
n=4 & 1 \quad 4 \quad 6 \quad 4 \quad 1 \\
n=5 & 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\
\vdots & \vdots
\end{array}$$

$$\sum_k \binom{a}{k+\phi}\binom{a}{k+\phi+2a+1}e^{i\frac{k+\phi}{3}\pi} = \binom{a}{-1/3}\frac{2\cos(a+\frac{1}{3})\pi - e^{i(a+2\phi)\pi}}{\left(3^{3/2}e^{i\pi/6}\right)^{a+1}}, \quad \Re a > -1.$$

$$\sum_k \binom{a}{3k+\phi}\binom{a}{3k+\phi+2a+1}(-1)^k = -\frac{\binom{a}{-\frac{a+1}{2}}\sin(\frac{1}{2}a+\phi)\pi}{3}$$

$$+\frac{2\binom{a}{-1/3}\left(2\sin(\phi-\frac{1}{6})\pi\sin(\frac{5}{6}a+\frac{2}{3}\phi)\pi-\sin(\frac{7}{6}a+\frac{1}{3}\phi)\pi\right)}{3^{(3a+5)/2}}, \qquad \Re a > -1.$$

$${}_4F_3\left[\begin{matrix} a, & \frac{-a}{2}, & \frac{1-a}{2}, & 1+\frac{a}{2e^{2i\pi/3}-1} \\ & \frac{2}{3}, & 1-a, & \frac{a}{2e^{2i\pi/3}-1}\end{matrix}\,\middle|\, \frac{4}{3\sqrt{3}}e^{-i\pi/6}\right] = e^{-ia\pi/3}.$$

$$\sum_k \frac{\prod_{n\geq 1} 1-s\tan\frac{2^m k+\phi}{(-2)^n}\pi}{(2^m k+\phi)^2} = \frac{\pi^2}{(\sin 2^{-m}\phi\pi)^2}\prod_{n=1}^{m}\frac{1-s\tan\frac{\phi\pi}{(-2)^n}}{4},$$

$$\sum_k \frac{\prod_{n\geq 1} 1-s\tan\frac{3k+\phi}{(-2)^n}\pi}{(3k+\phi)^2} = \frac{\pi^2}{s^2+9}\,\frac{s^2\frac{\cos\phi\pi}{\cos\frac{\phi\pi}{3}}+4s\sin\frac{\phi\pi}{3}+3\frac{\sin\phi\pi}{\sin\frac{\phi\pi}{3}}}{3\sin\phi\pi\sin\frac{\phi\pi}{3}}.$$

$$\sum_{k\neq 0}\frac{(-2)^{\alpha_2(k)}}{k^2}\prod_{n\geq 1} 1+s\tan\frac{k\pi}{(-2)^{n+\alpha_2(k)+1}} = \begin{cases} \frac{\pi^2}{6}, & |s|<\sqrt{3}; \\ 0, & |s|=\sqrt{3}; \\ \nexists, & |s|>\sqrt{3}.\end{cases}$$

$$k =: 2^{\alpha_2(k)}3^{\alpha_3(k)}5^{\alpha_5(k)}\ldots$$

$$\sum_{k>-\infty}\frac{\prod_{n\geq 2} 1+\tan\frac{2k+1}{(-2)^n}\pi}{(2k+1)^2}e^{i\pi k/82} = \frac{\pi^2}{e^{i\pi/164}}\left(\frac{5}{22}+\frac{i}{62}\right),$$

$$\sum_{k>-\infty}\frac{\prod_{n\geq 2} 1+\tan\frac{2k+1}{(-2)^n}\pi}{(2k+1)^2}e^{i\pi k/130} = \frac{\pi^2}{e^{i\pi/260}}\frac{2}{9},$$

$$\sum_{k>-\infty}\frac{\prod_{n\geq 2} 1+\tan\frac{2k+1}{(-2)^n}\pi}{(2k+1)^2}e^{i\pi k/131} = \frac{\pi^2}{e^{i\pi/262}}\,\frac{32643412914226218842+i23873079169898386 9}{2^{67}+2^{35}+4}.$$

$$1 - z + z^2 = \frac{1 + z^2 + z^4}{1 + z + z^2} = \frac{1 + z^3}{1 + z}$$

$\Longrightarrow$

$$\prod_{n \geq 1} \left(2\cos\left(\frac{\theta}{2^n}\right) - 1 \right) = \frac{1 + 2\cos\theta}{3},$$

$$\prod_{n \geq 1} \left(2\cos\left(\frac{\theta}{3^n}\right) - 1 \right) = \cos\frac{\theta}{2},$$

$$\prod_{n \geq 1} \left(1 + 2\sin\frac{\theta}{(-3)^n} \right) = \cos\frac{\theta}{2} - \sin\frac{\theta}{2};$$

$$\prod_{n \geq 1} \frac{1 + 2\cos\frac{\theta}{3^n}}{3} = \frac{2}{\theta}\sin\frac{\theta}{2}$$

$$\prod_{n \geq 1} \frac{1 + 2\cos\frac{\theta}{3^{n/2}}}{3} = \frac{4}{\theta^2\sqrt{3}}\sin\frac{\theta}{2}\,\sin\frac{\theta\sqrt{3}}{2}$$

$$\prod_{n\geq 1} f\left(\frac{\theta}{a^n}\right) = g(\theta)$$

$\Longrightarrow$

$$\int_0^{\infty} \ln f\left(\frac{\theta}{a^n}\right) dn = \int_0^1 \ln g(a^n\theta)\, dn$$

$\Longleftrightarrow$

$$\int_0^{\theta} \frac{\ln f(x)}{x}\, dx = \int_{\theta}^{a\theta} \frac{\ln g(x)}{x}\, dx \qquad e.g.,$$

$$\int_0^{\theta} \frac{\ln(1+2\cos x) - \ln 3}{x}\, dx = \int_{\theta}^{3\theta} \frac{\ln\sin\frac{x}{2}}{x}\, dx - \ln 3 \ln \frac{\theta\sqrt{3}}{2}$$

$$\prod_{k,n\geq 1} \left(2\cos\left(\frac{\theta}{2^k 3^n}\right) - 1\right) = \frac{2}{\theta}\sin\frac{\theta}{2}$$

$\Longrightarrow$

$$\int_0^{\alpha}\int_0^{\beta} \frac{\ln\left(2\cos(uv) - 1\right)}{uv}\, du dv = \int_{\alpha}^{2\alpha}\int_{\beta}^{3\beta} \frac{\ln\sin\frac{uv}{2}}{uv}\, du dv$$

$$- \ln 2 \ln 3 \ln\left(\alpha\beta\sqrt{\frac{3}{2}}\right), \qquad |\alpha\beta| \leq \frac{\pi}{3}$$

$\Longrightarrow$

$$(a_0 + a_1 z + a_2 z^2)(b_0 + b_1 z + b_2 z^2) = a_2 + a_1 z^2 + a_0 z^4, \qquad \forall z$$

$$\prod_{n\geq 1} 2\sin\left(\frac{\theta}{(-2)^n} + \frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}\cos\left(\theta + \frac{\pi}{6}\right),$$

$$\prod_{n\geq 1} \frac{1}{2} + \sin\left(\frac{\theta}{(-2)^n} + \frac{\pi}{6}\right) = \frac{2\sin\left(\theta + \frac{\pi}{6}\right) - 1}{\theta\sqrt{3}}.$$

$$\prod_{n\geq 1} 4\cos\left(\frac{\theta}{(-4)^n} + \frac{\pi}{5}\right)\cos\left(\frac{2\theta}{(-4)^n} + \frac{2\pi}{5}\right) = \frac{\sin\left(\theta + \frac{\pi}{5}\right)}{\sin\frac{\pi}{5}}.$$

$$\Longrightarrow \qquad \ln z \approx 6\frac{z-1}{1+4\sqrt{z}+z}$$

$$\frac{z-1}{1}-\frac{(z-1)^2}{2}+\frac{(z-1)^3}{3}-\frac{(z-1)^4}{4}+\frac{(z-1)^5}{5}-\cdots$$

$$\approx \frac{z-1}{1}-\frac{(z-1)^2}{2}+\frac{(z-1)^3}{3}-\frac{(z-1)^4}{4}+\frac{575}{576}\frac{(z-1)^5}{5}-\cdots$$

$$\ln z = \frac{z-1}{\displaystyle\int_0^1 z^t\,dt}$$

$$\frac{6}{p}\frac{z^{2p}-1}{z^{2p}+4z^p+1}=\frac{z-1}{1}-\cdots-\frac{(z-1)^4}{4}+\left(1-\frac{p^4}{36}\right)\frac{(z-1)^5}{5}-\cdots.$$

$$T(t):=\sum_k \frac{\displaystyle\prod_{n\geq 1} 1+z\left(\sin\frac{k+\phi}{3^n}\pi\right)^2}{(k+\phi)\pi}e^{i\pi(k+\phi)(2t-1)}.$$

$$T(2/13)=\cot\tfrac{\phi\pi}{13}+\frac{4i}{z^2+4},$$

The complex parameter z is the first and last step of the zigzag whose middle step is $-4-2z$. The dimension D is determined by $2|z/4|^D+|1+z/2|^D=1$.

$$\sum_k \frac{1-\displaystyle\prod_{n\geq 1} 1+z\left(\sin\frac{ak+b}{3^n}\pi\right)^2}{ak+b}=0, \qquad -2\leq a\leq 2$$

$$f\pm(a,b,z):=\sum_k(\pm)^k\frac{1-\displaystyle\prod_{n\geq 1} 1+z\left(\sin\frac{ak+b}{3^n}\pi\right)^2}{ak+b},$$

(so that $f_+(a,b,z)+f_-(a,b,z)=f_+(2a,b,z)$), then

$$f\pm(1,b,z)=0,$$
$$f_+(2,b,z)=0,$$

but

$$f_+(3,b,z) = -\frac{\pi z}{6}\sin\frac{2b\pi}{3}$$
$$f_-(3,b,z) = -\frac{\pi z}{3}\sin\frac{b\pi}{3}$$
$$f_+(4,b,z) = \frac{\pi z}{z-4}\sin\frac{b\pi}{2}$$
$$f_+(5,b,z) = \frac{4\pi z}{5}\frac{z\sin\frac{4b\pi}{5} - 4\sin\frac{2b\pi}{5}}{z^2+16}$$
$$f_-(5,b,z) = \frac{4\pi z}{5}\frac{\sin\frac{2b\pi}{5} + 2\sin\frac{b\pi}{5}}{z-2}$$
$$\vdots$$
$$f_-(13,b,z) = \frac{2\pi z}{13}\left(2\frac{z(\sin\frac{11b\pi}{13} - 4\sin\frac{5b\pi}{13}) - 4(\sin\frac{7b\pi}{13} + \sin\frac{5b\pi}{13})}{z^2-4z+16} - \frac{z(\sin\frac{9b\pi}{13} + \sin\frac{3b\pi}{13}) - 2\sin\frac{3b\pi}{13} - 4\sin\frac{b\pi}{13}}{z^2+4}\right)$$

$$f(t) = \sum_{|k|\le\nu} a_{2k+1}e^{(2k+1)i\pi t}, \qquad a_k := (1\ 0\ 0)\left(\prod_{n\ge1} M(k\pi/2^n)\right)\begin{pmatrix}0\\0\\1\end{pmatrix},$$

$$M(\theta) := \frac{\sec\alpha}{4}\begin{pmatrix} e^{-i\alpha} & -e^{i(\alpha-\theta)} & 2i\sin\left(\alpha+\frac{\theta}{2}\right)e^{-i\theta/2} \\ -e^{i\alpha} & e^{-i(\alpha+\theta)} & 2i\sin\left(\alpha-\frac{\theta}{2}\right)e^{-i\theta/2} \\ 0 & 0 & 4\cos\alpha\cos\frac{\theta}{2}e^{-i\theta/2}\end{pmatrix}.$$

$$n\,! = 1 \cdot 2 \cdot 3 \cdots n.$$

$$n\,!_{x} := 1^{1^x} 2^{2^x} 3^{3^x} \ldots n^{n^x}.$$

$$\begin{aligned} z\,!_{1} \left(\frac{2\pi}{e^{z+1}}\right)^{z/2} &= e^{\int_0^z \ln t!\,dt} \\ &= z!^{z/2} \prod_{n\ge 1} \frac{e^z}{\left(1+\dfrac{z}{n}\right)^{n+z/2}}. \end{aligned}$$

$$\begin{aligned} \pi_0 &:= \left(-\frac{1}{2}\right)!^2 = \pi; \\ \pi_1 &:= \left(-\frac{1}{2}\right)!_{1}^{\,2} = \sqrt[6]{2}\,\sqrt[4]{\pi e^{\gamma-1-\zeta'(2)/\zeta(2)}}; \\ \pi_2 &:= \left(-\frac{1}{2}\right)!_{2}^{\,2} = e^{7\zeta(3)/8\pi^2}; \\ &\ \vdots \end{aligned}$$

$$\begin{aligned} \Psi_{-1}(z+1) &:= \ln z! + c_{-1}; \\ \Psi_{-2}(z+1) &:= \int_0^z \ln t!\,dt + c_{-1}x + c_{-2}; \\ \ln z\,!_{1} &= \int_0^z \ln t!\,dt + z\,\frac{z+1-\ln 2\pi}{2}; \\ \Psi_{-3}(z+1) &:= \int_0^z (z-t)\ln t!\,dt + c_{-1}x^2 + c_{-2}x + c_{-3}; \\ \ln z\,!_{2} &= 2\int_0^z (z-t)\ln t!\,dt + \frac{(z+1)(z+1/2)z}{8} - \frac{z}{6}\left(\gamma - \frac{\zeta'(2)}{\zeta(2)} + (3z+1)\ln 2\pi\right). \end{aligned}$$

$$\lambda := \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - \cdots = 2\pi \ln \frac{(-3/4)\,!_{1}}{(-1/4)\,!_{1}}.$$

$$\mathrm{Li}_2(e^{2\pi i z}) = \pi^2 B_2(z) + 2\pi i \ln \frac{(z-1)!_1}{(-z)!_1}.$$

$$\prod_{j=0}^{k-1} \left(z - \frac{j}{k}\right)!_n = \left(\frac{(kz)!_n \left(2^{\frac{B_{n+1}(1)}{n+1}} \pi_n^{2^{n-1}}\right)^{\frac{k^{n+1}-1}{2^{n+1}-1}}}{k^{\frac{B_{n+1}(1+kz)}{n+1}}} \right)^{k^{-n}}.$$

$$\prod_{n \geq 1} \frac{e^{\frac{1}{12} - n}}{\left(1 - \frac{1}{2n}\right)^{2n(n-\frac{1}{3})}} = \frac{e^{\frac{7\zeta(3)}{48\zeta(2)}}}{2^{\frac{5}{36}}} = 1.0103523768335617924 75+.$$

$$\lim_{\substack{n \to n_0 \\ k \to k_0}} \binom{-n}{k} = k$$

$$y = 0.662743419349181+ \quad \Longleftrightarrow \quad y \sinh \sqrt{1+y^2} = 1.$$

$$\lim_{\substack{n \to n_0 \\ k \to k_0}} \binom{-n}{k} = \text{ķ}$$

puķe

(A sickening shade of grayish purple, popularly known as the Dreadful Grape)

Mann muß immer \unkern.

— Felix Knuth

$$y = 0.662743419349181+ \quad \Longleftrightarrow \quad y \sinh \sqrt{1+y^2} = 1\,.$$

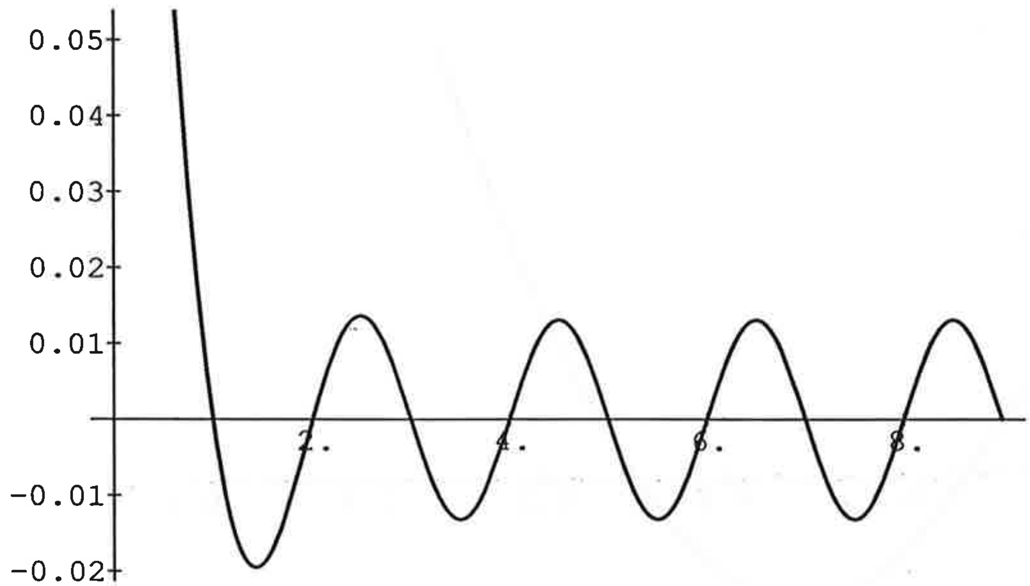
= radius of convergence of continued cosine power series

= y intercepts of next two plots

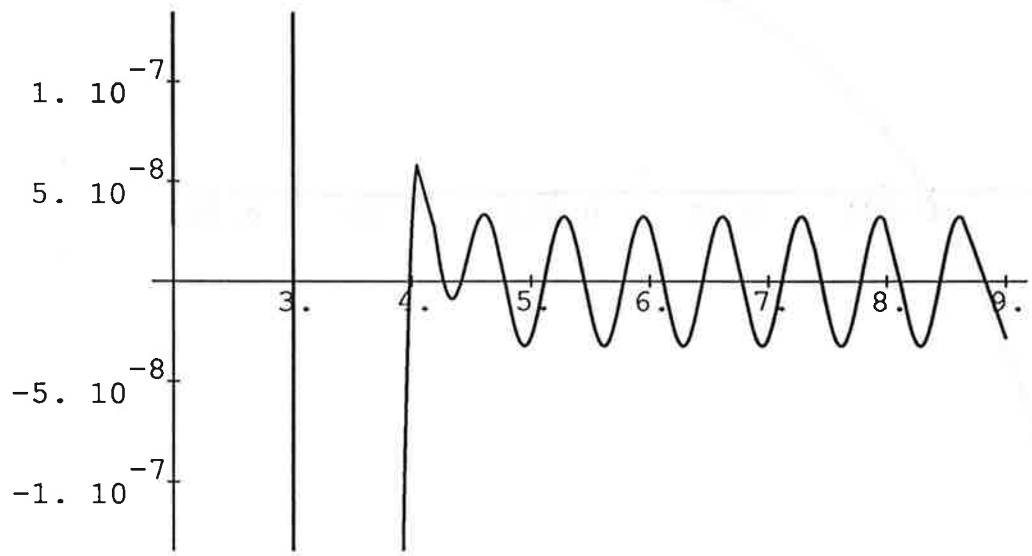
radius of convergence

"Always invert" (Jacobi)

~~b18~~

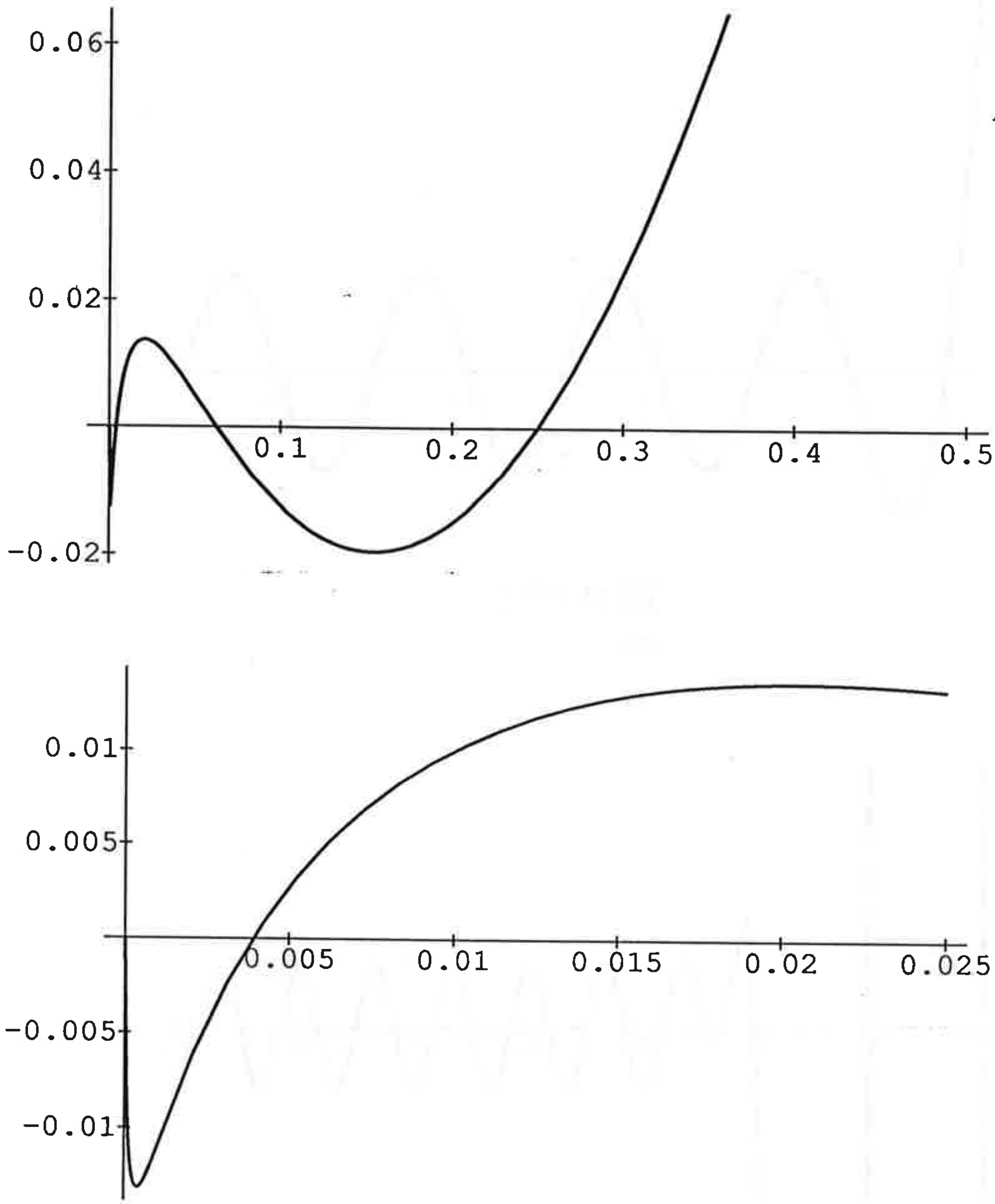


$$\prod_{k\geq 1} 2^{1-2^{x-k}}-1$$



$$\prod_{k\geq 1} (2^{1-2^{x-k}}-1) - 0.01311359272 \sin(\pi x - .00000216572)$$

~~a18~~



$$\prod_{k\geq 1} 2t^{2^{-k}}-1$$

$$(-)^{n+1}n\binom{x}{n}\prod_{k=0}^{n}\begin{pmatrix}\frac{k-n+1}{k+2} & (k+1)^p & 0\\ 0 & \frac{k-n}{k+1} & \frac{x-n}{x-k}\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}0 & 0 & \frac{B_{p+1}(x+1)-\dot{B}_p}{p+1}\\ 0 & 0 & -n\\ 0 & 0 & (-)^{n+1}n\binom{x}{n}\end{pmatrix},$$

$$n\prod_{k=1}^{n}\begin{pmatrix}\frac{k-n}{k+1} & \frac{k-n}{k+1}k^p & k^{p-1}\\ 0 & \frac{k-n}{k+1} & \frac{1}{k}\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}0 & 0 & B_p(1)\\ 0 & 0 & H_n\\ 0 & 0 & n\end{pmatrix},$$

i.e.,

$$\sum_{k=1}^{n}\frac{(-)^{k-1}}{k}\binom{n}{k}\sum_{j=1}^{k}j^p=B_p(1), \qquad \forall n=\lfloor n\rfloor>p=\lfloor p\rfloor$$

$$\frac{\prod_{k\ge1}2t^{2^{-k}}-1}{t\ln t}$$

$$n!=1\cdot2\cdot3\cdots n..$$

$$n\,\underset{x}{!}:=1^{1^x}2^{2^x}3^{3^x}\ldots n^{n^x}. \qquad 1^{1^x}2^{2^x}3^{3^x}\cdots n^{n^x}$$

$$z\,\underset{1}{!}\left(\frac{2\pi}{e^{z+1}}\right)^{z/2}=e^{\int_0^z\ln t!\,dt}$$

$$=z!^{z/2}\prod_{n\ge1}\frac{e^z}{\left(1+\frac{z}{n}\right)^{n+z/2}}.$$

$$\pi_0 := \left(-\frac{1}{2}\right)!^2 = \pi;$$

$$\pi_1 := \left(-\frac{1}{2}\right)!_1^2 = \sqrt[6]{2}\sqrt[4]{\pi e^{\gamma-1-\zeta'(2)/\zeta(2)}};$$

$$\pi_2 := \left(-\frac{1}{2}\right)!_2^2 = e^{7\zeta(3)/8\pi^2};$$

$$\vdots$$

$$\Psi_{-1}(z+1) := \ln z! + c_{-1};$$

$$\Psi_{-2}(z+1) := \int_0^z \ln t!\, dt + c_{-1}x + c_{-2};$$

$$\ln z!_1 = \int_0^z \ln t!\, dt + z\,\frac{z+1-\ln 2\pi}{2};$$

$$\Psi_{-3}(z+1) := \int_0^z (z-t)\ln t!\, dt + c_{-1}x^2 + c_{-2}x + c_{-3};$$

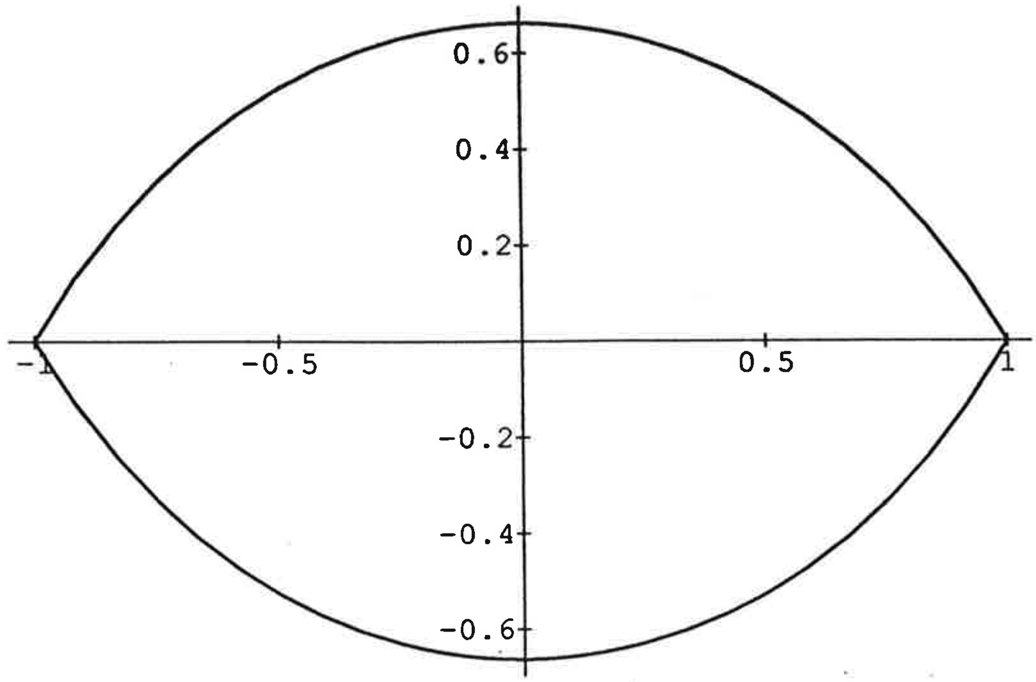
$$\ln z!_2 = 2\int_0^z (z-t)\ln t!\, dt + \frac{(z+1)(z+1/2)z}{8} - \frac{z}{6}\left(\gamma - \frac{\zeta'(2)}{\zeta(2)} + (3z+1)\ln 2\pi\right).$$

$$\lambda := \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - \cdots = 2\pi \ln \frac{(-3/4)!_1}{(-1/4)!_1}.$$

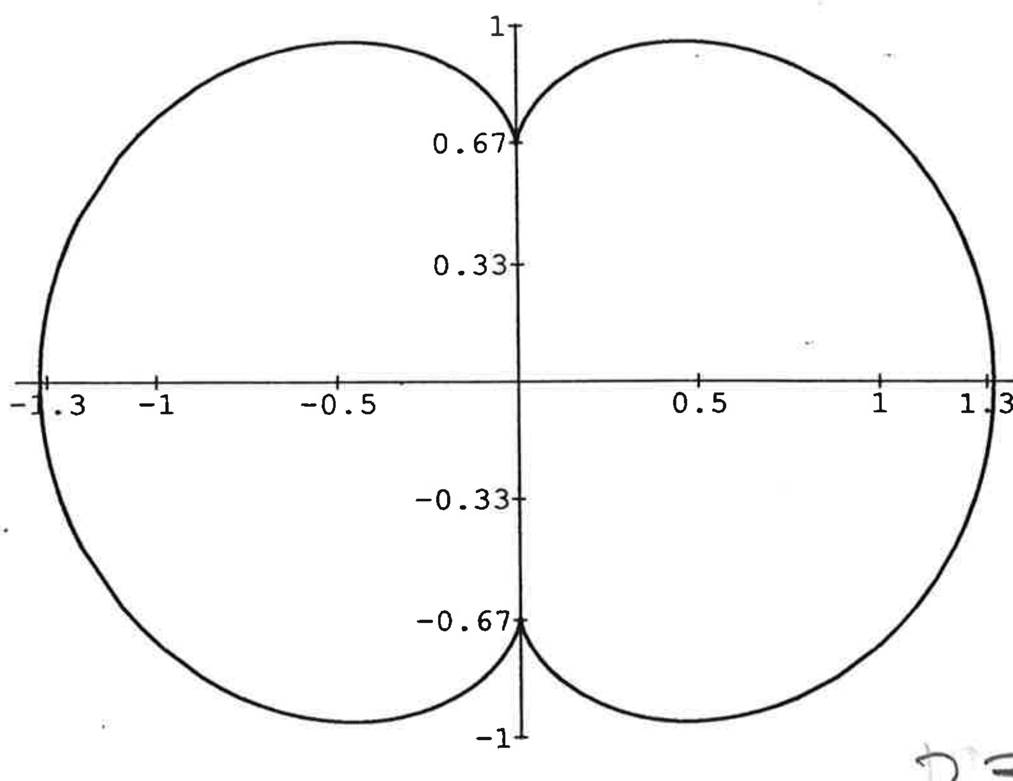
$$\mathrm{Li}_2(e^{2\pi i z}) = \pi^2 B_2(z) + 2\pi i \ln \frac{(z-1)!_1}{(-z)!_1}.$$

$$\prod_{j=0}^{k-1}\left(z - \frac{j}{k}\right)!_n = \left(\frac{(kz)!_n \left(2^{\frac{B_{n+1}(1)}{n+1}}\pi_n^{2^{n-1}}\right)^{\frac{k^{n+1}-1}{2^{n+1}-1}}}{k^{\frac{B_{n+1}(1+kz)}{n+1}}}\right)^{k^{-n}}.$$

$$\prod_{n\geq 1}\frac{e^{\frac{1}{12}}-n}{\left(1-\frac{1}{2n}\right)^{2n(n-\frac{1}{3})}} = \frac{e^{\frac{7\zeta(3)}{48\zeta(2)}}}{2^{\frac{5}{36}}} = 1.0103523768335617924 75+.$$



$\lim_{n \to \infty} (|J_n(nz)| = 1)$



$|z \sin z_0| = 1, \quad z_0 = z \cos z_0$

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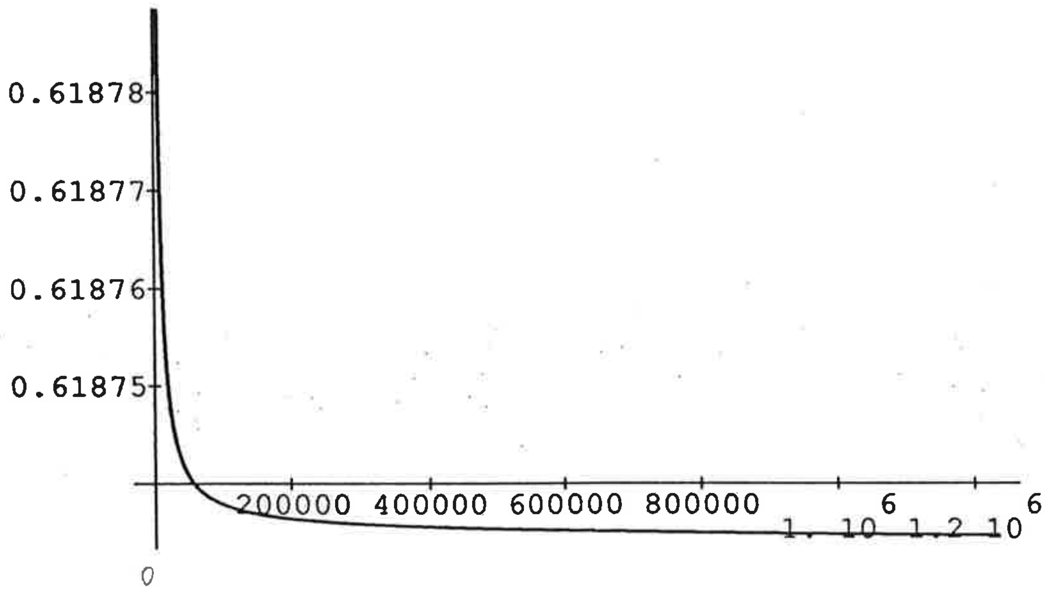
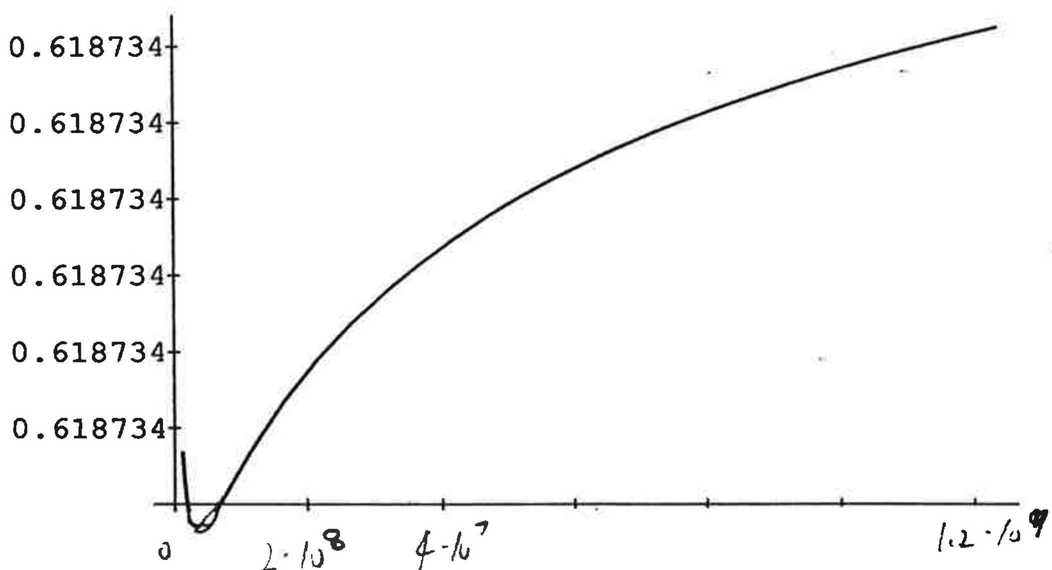
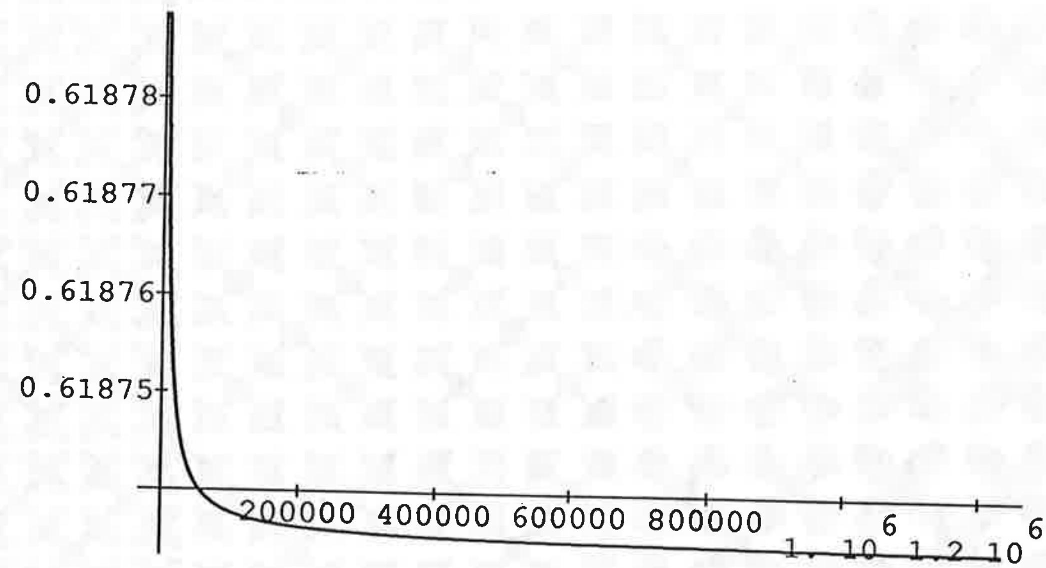
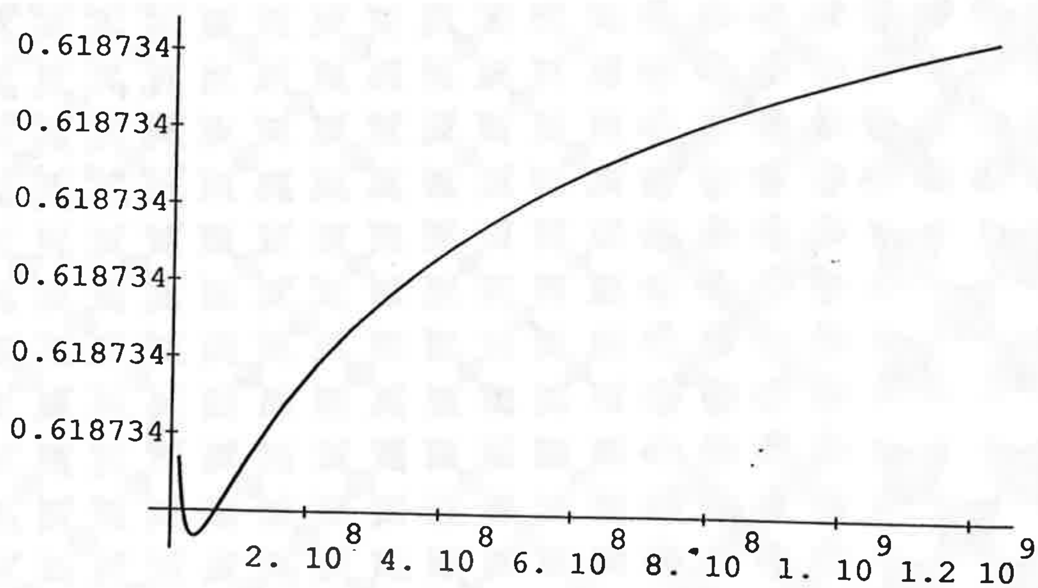




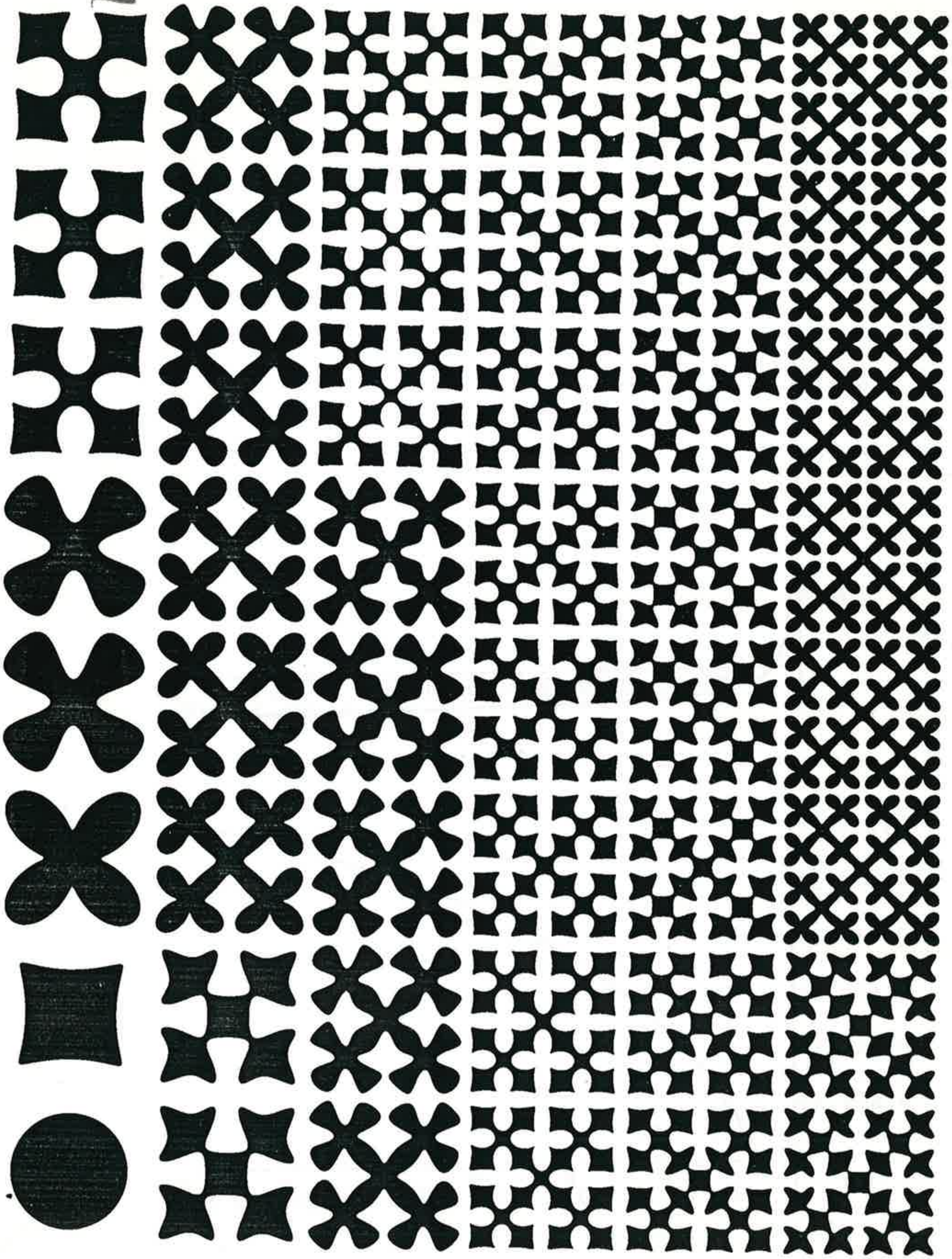
$$\frac{\prod_{k\geq 1} 2\sqrt[2^k]{t}-1}{t\ln t}$$

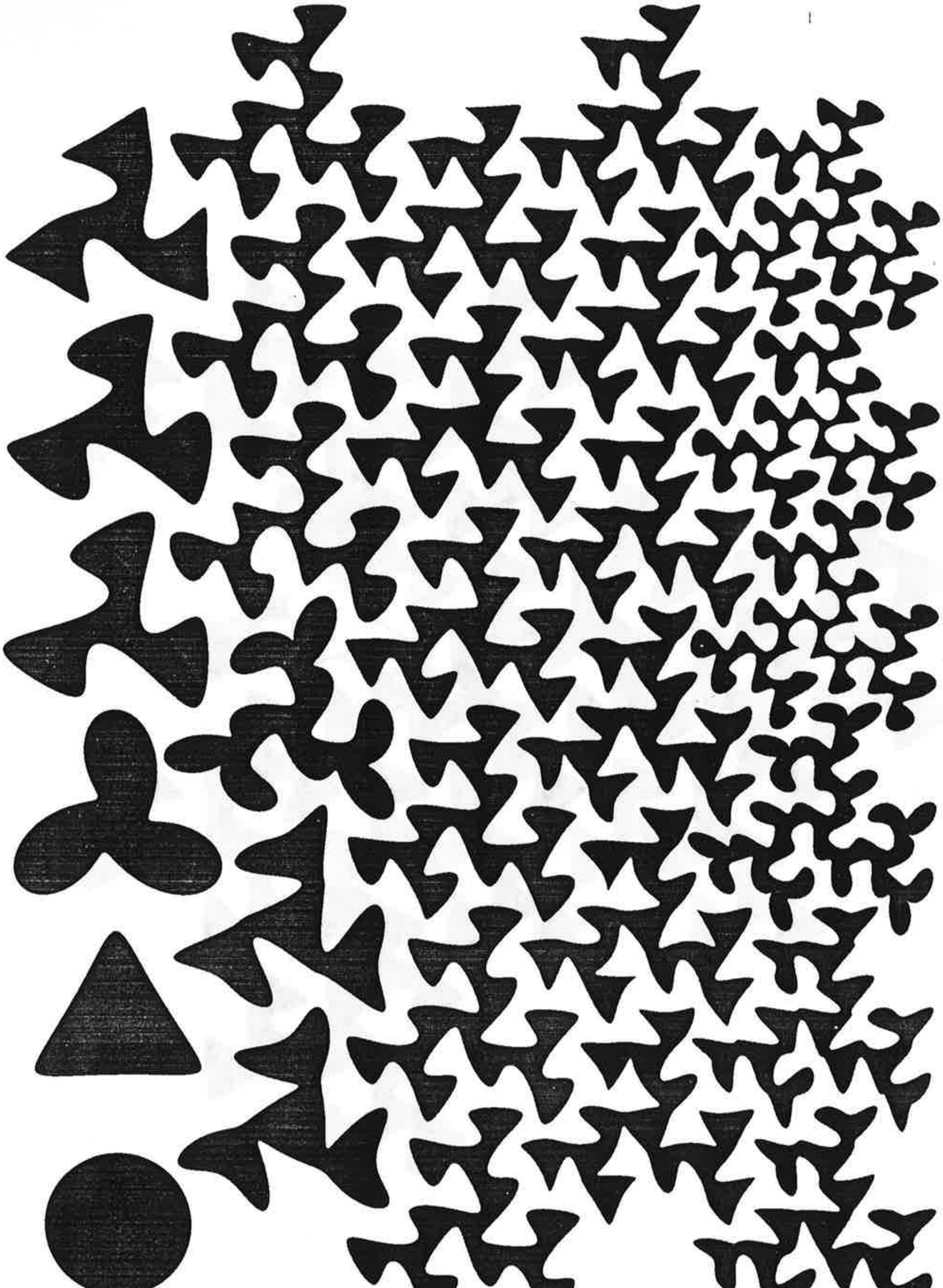
$$\frac{\prod_{k\geq 1} 2t^{2^{-k}} - 1}{t \ln t}$$

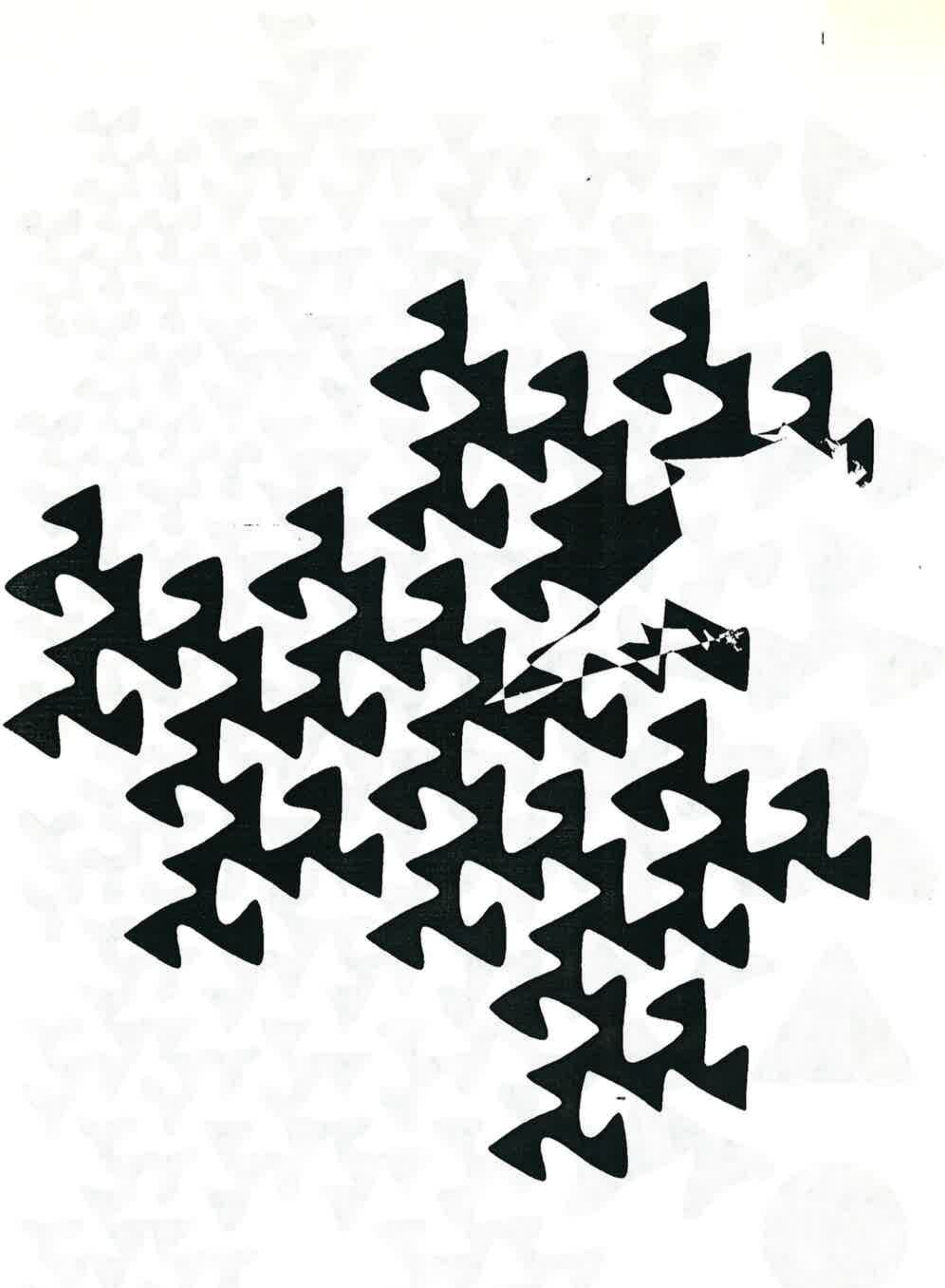




The numbering of the remaining pages is fairly arbitrary. NJAS







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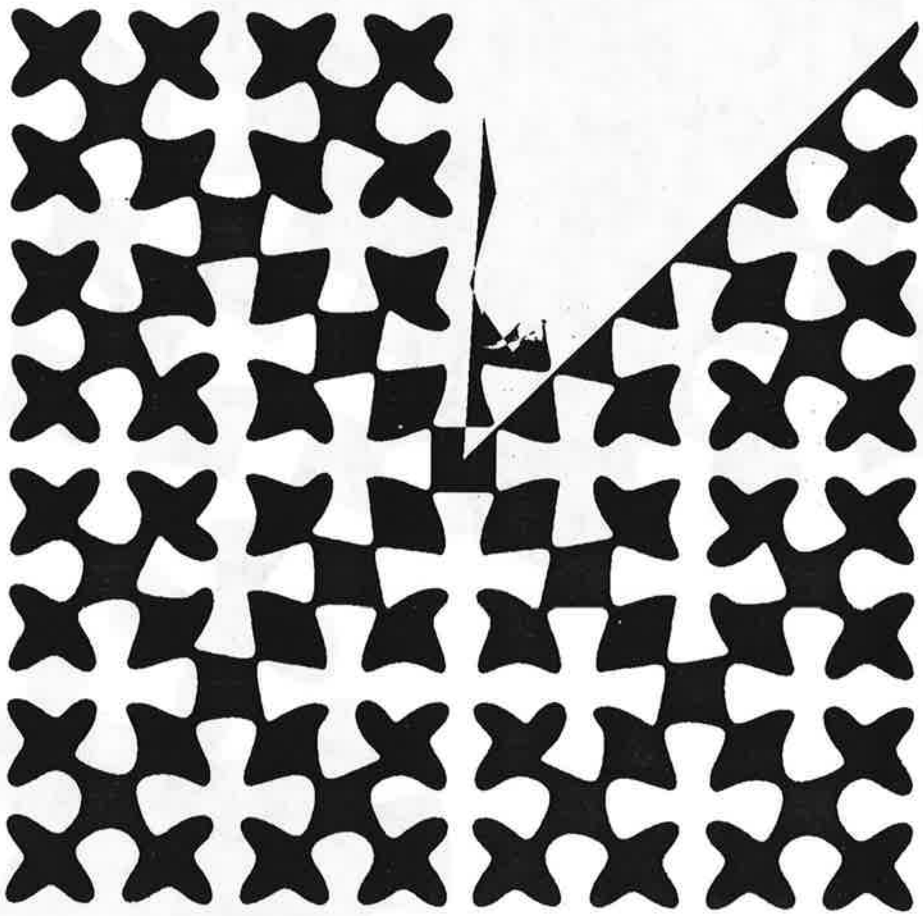

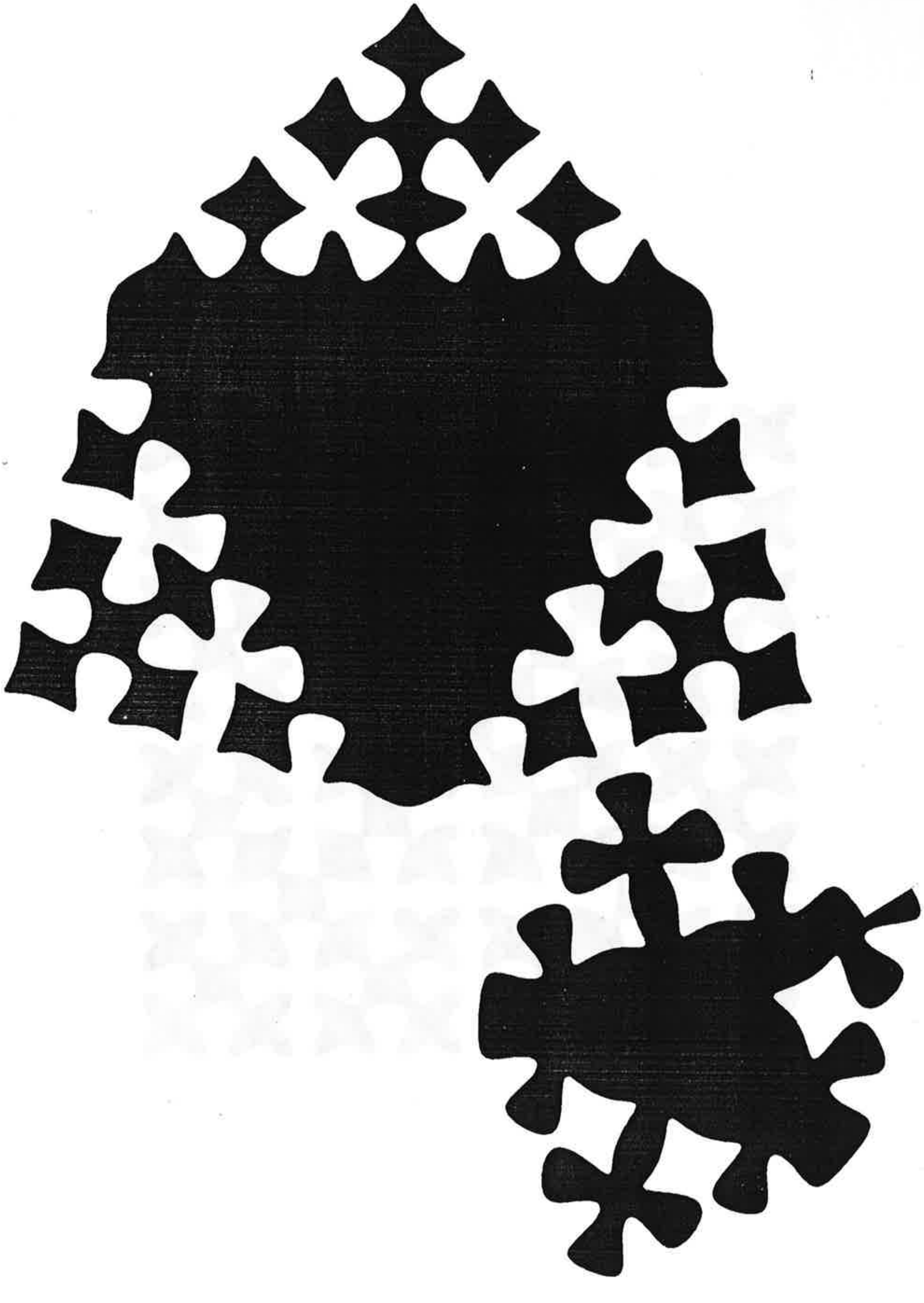

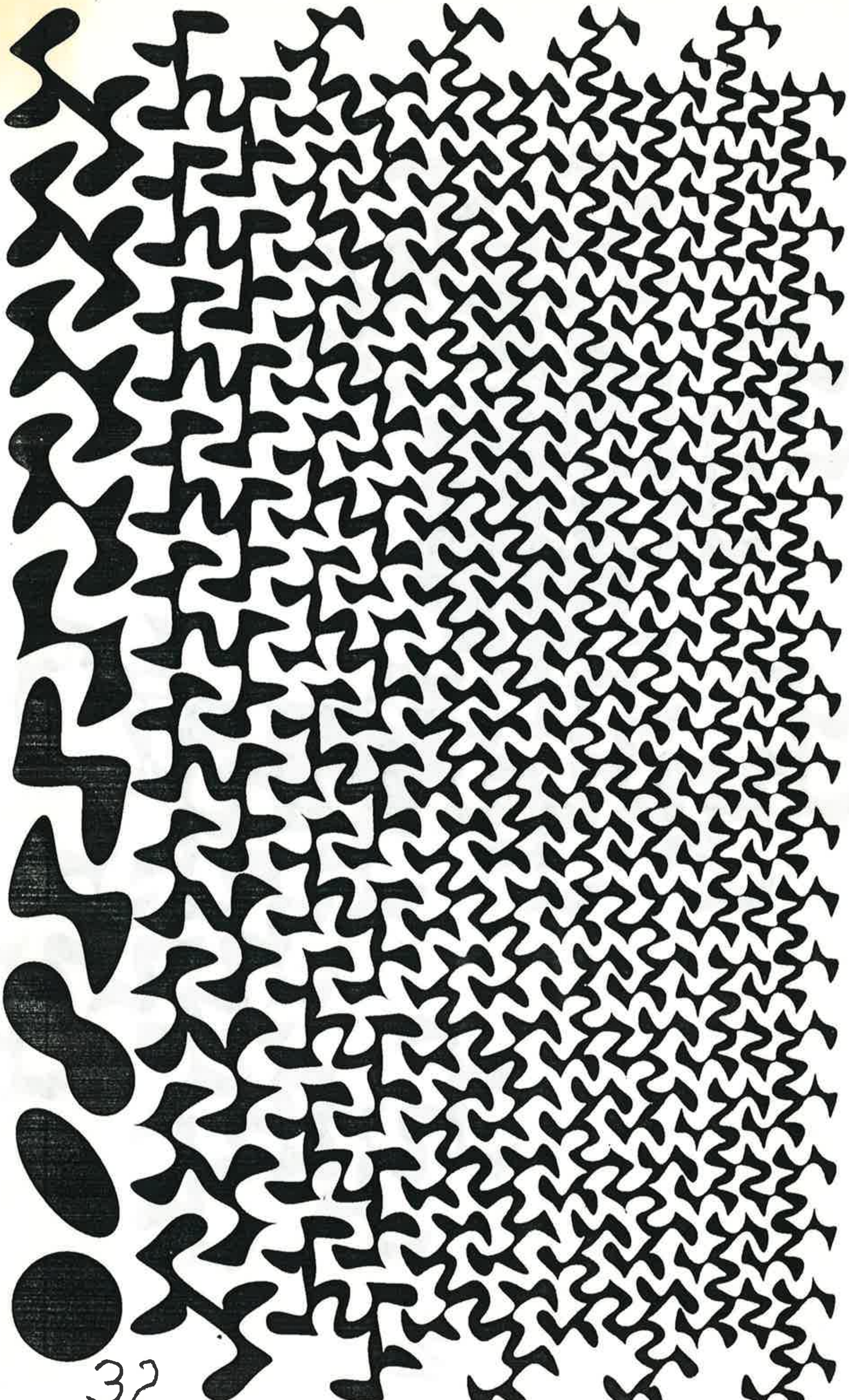

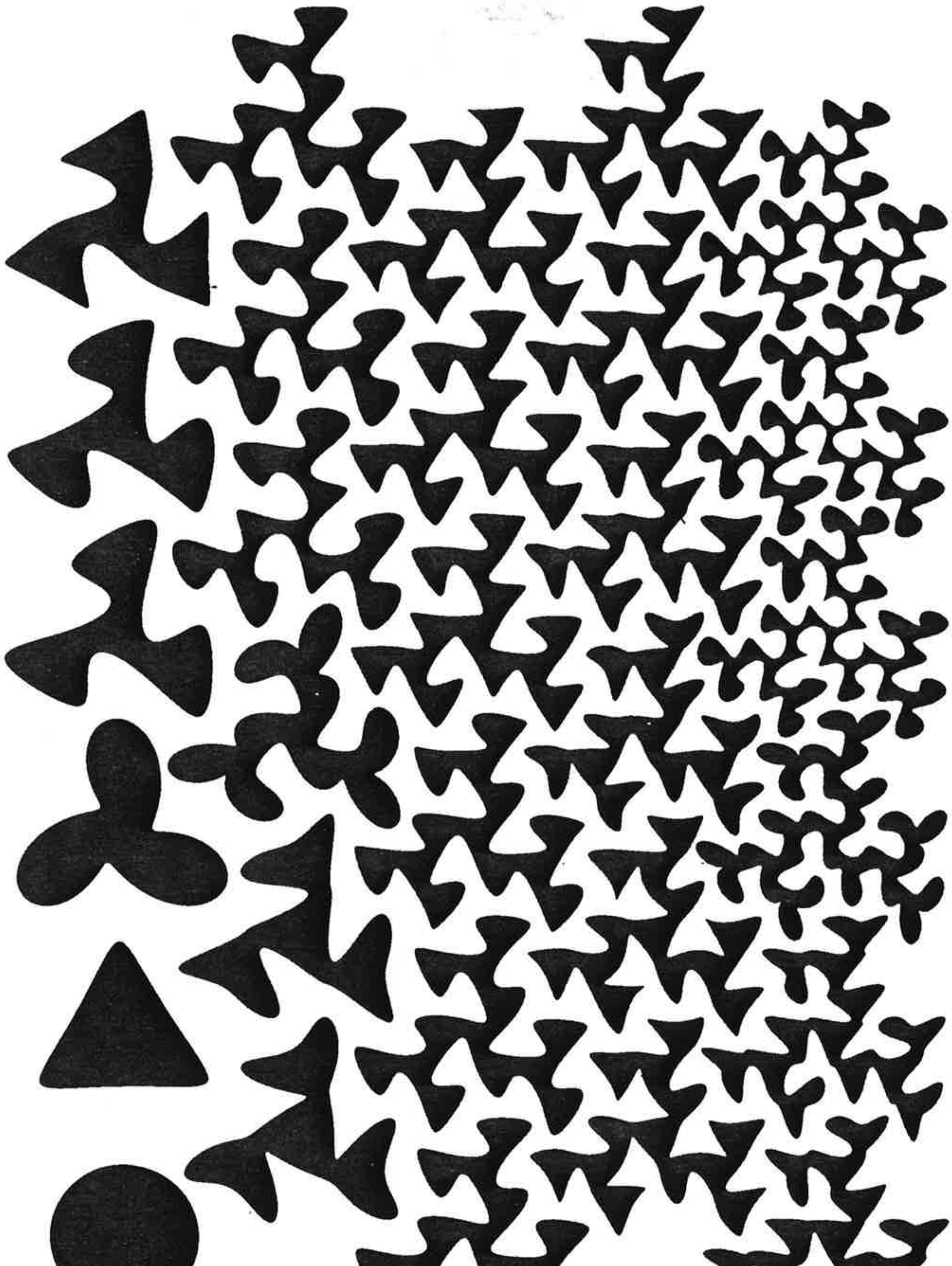

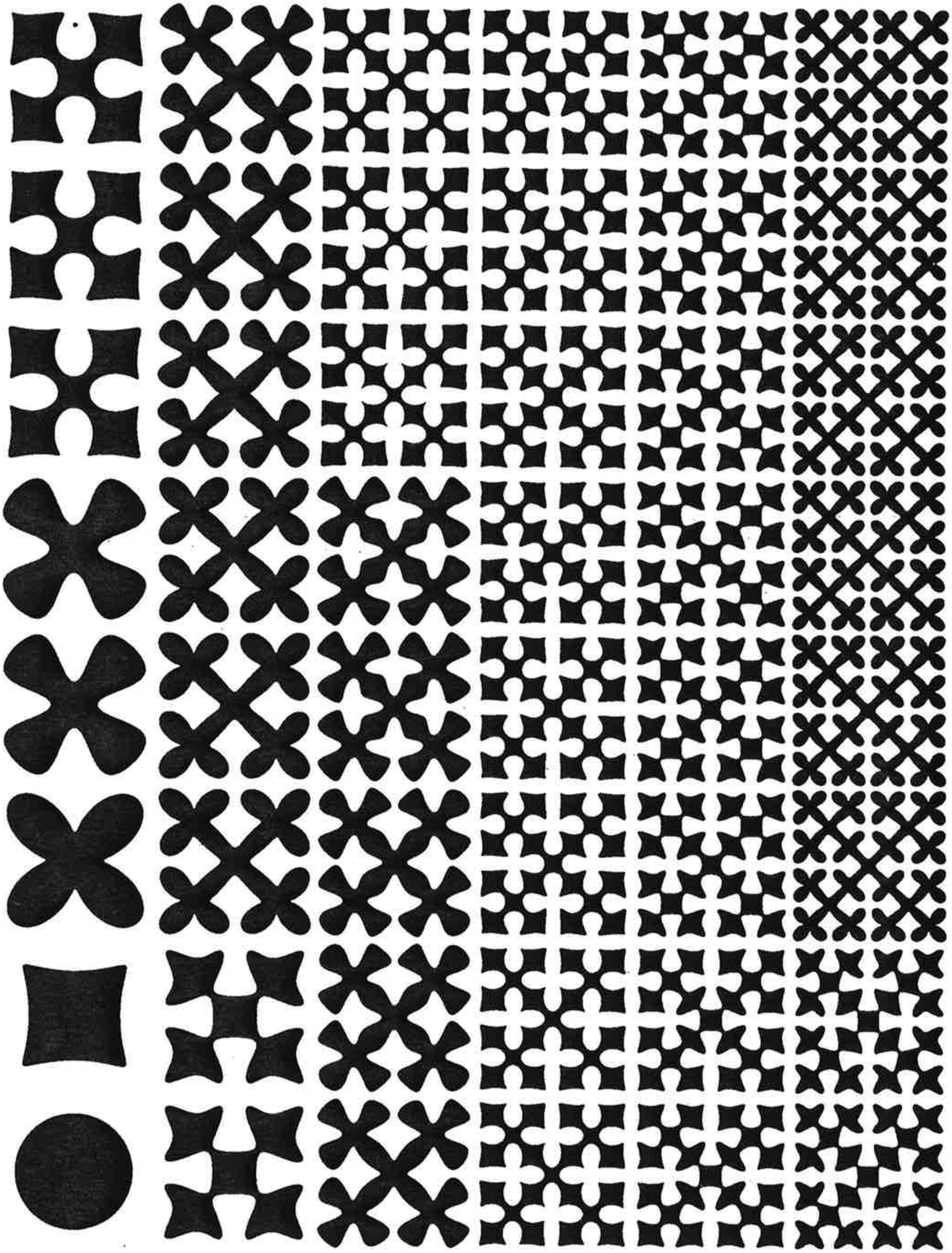

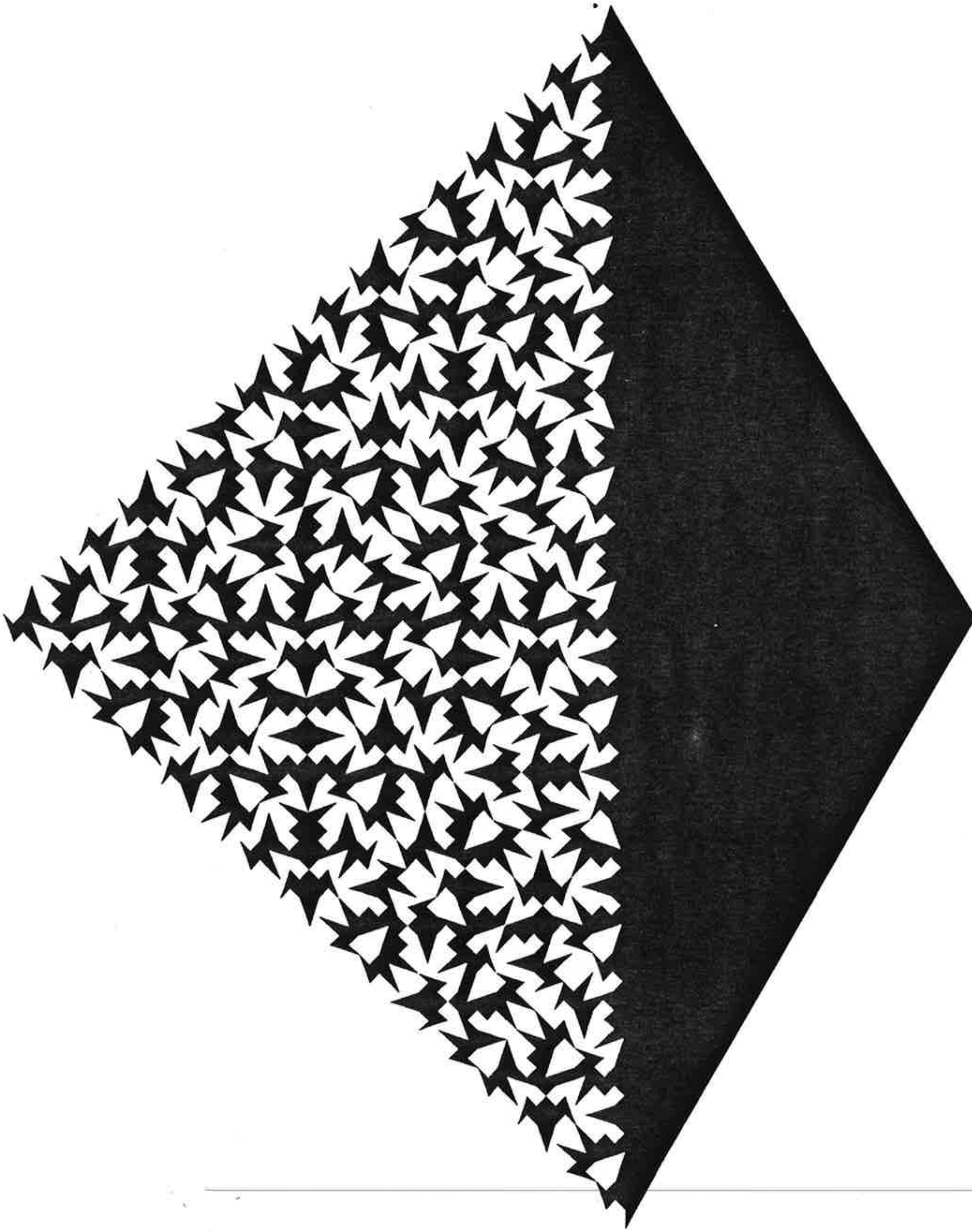

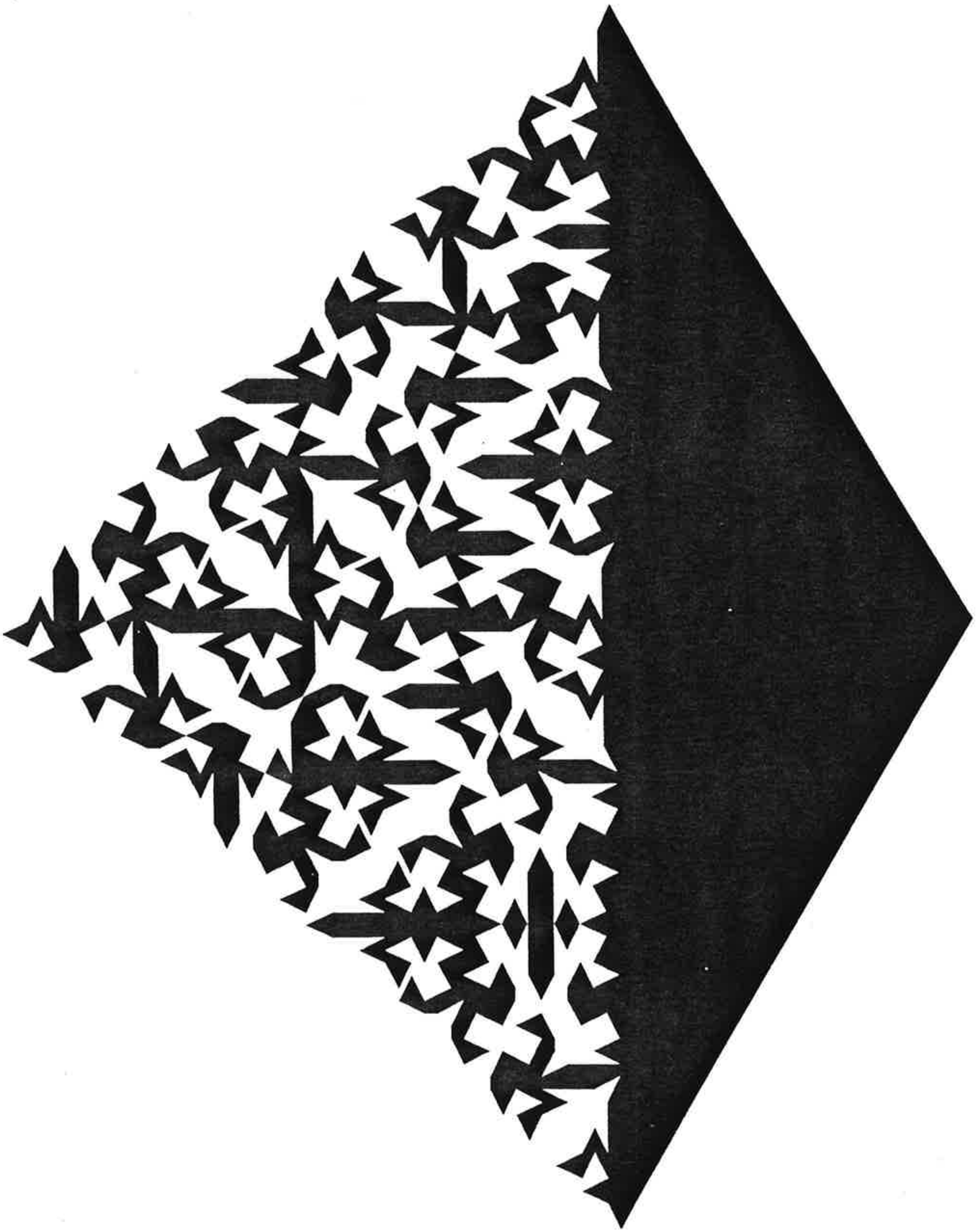

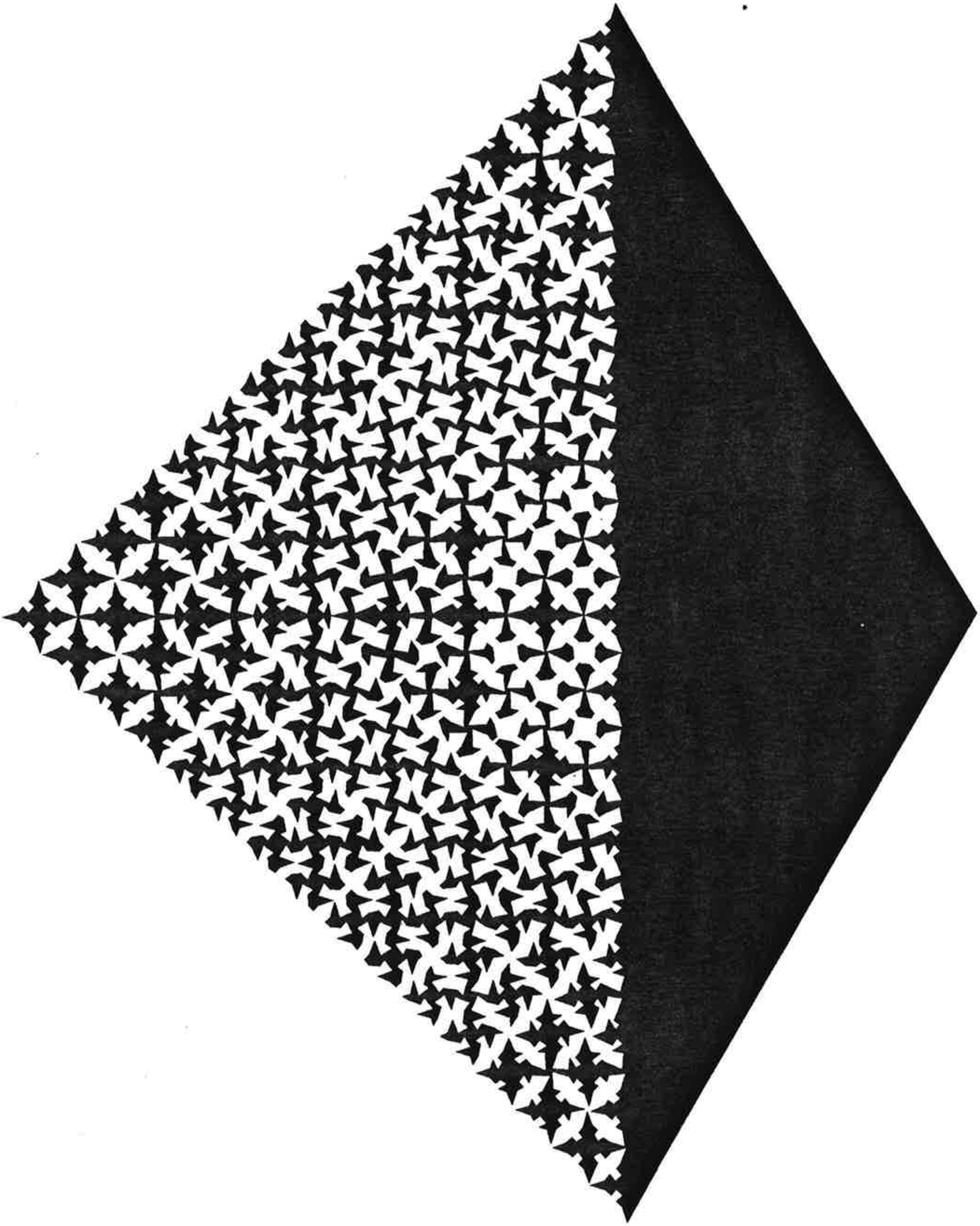

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