Proof of the equivalences in the Comment section

Hartmut F. W. Hoft, Nov 04, 2022

Proof of equivalence of (1) and (2): If n = m*k with m <= k < 2*m then sqrt(n/2) < m < sqrt(2*n), proving (2). If divisor d|n satisfies sqrt(n/2) < d < sqrt(2*n) and n = d*e then either e < d < 2*e or d < e < 2*d so that the pair (d, e) or the pair (e, d) are members of the set in (1).

Proof of equivalence of (2) and (3):
Suppose n = d*e and sqrt(n/2) < d < sqrt(2*n) then also sqrt(n/2) < e < sqrt(2*n). For any factoring n = x*y the inequalities r(n) < x < sqrt(2*n) lead to the contradiction 2*y < x + 1 < 2*y + 1. Therefore, d, n/d <= r(n) hold, which proves (3).
Suppose that d|n, that d, n/d ≤ r(n) < sqrt(2*n), and that d <= sqrt(n/2), then n = d*(n/d) < sqrt(n/2)*sqrt(2*n) = n, a contradiction. Therefore, sqrt(n/2) < d and similarly sqrt(n/2) < n/d proving (2).