Relation between A293670 and A002024
Numeric structure of an anamorphosis

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Purpose

This document was created to accompany my post on the OEIS for new sequence A293670 :

									1	-1									
									0	2									
								1	0	2	-1								
								1	2	0	3								
							1	1	2	0	3	-1							
							2	2	1	3	0	4							
						1	2	2	1	3	0	4	-1						
						3	2	2	3	1	4	0	5						
					1	3	2	2	3	1	4	0	5	-1					
					4	2	3	3	2	4	1	5	0	6					
				1	4	2	3	3	2	4	1	5	0	6	-1				
				5	2	4	3	3	4	2	5	1	6	0	7				
			1	5	2	4	3	3	4	2	5	1	6	0	7	-1			
			6	2	5	3	4	4	3	5	2	6	1	7	0	8			
		1	6	2	5	3	4	4	3	5	2	6	1	7	0	8	-1		
		7	2	6	3	5	4	4	5	3	6	2	7	1	8	0	9		
	1	7	2	6	3	5	4	4	5	3	6	2	7	1	8	0	9	-1	
	8	2	7	3	6	4	5	5	4	6	3	7	2	8	1	9	0	10	

etc.

The sequence is built the following way :

- start with an empty row (assimilable to row number 0)
- for all n >= 1, row n is obtained from row n-1, with
  - if n is odd : append 1 to the left and append -1 to the right
  - if n is even : replace all values by their complement to n/2.

Example of implementation :

```
(PARI code)
evolve(L,n)=if(n%2==1,listinsert(L,1,1);listinsert(L,-1,#L+1),L=apply(v->n/2-v,L));L
N=30;L=List();for(n=1,N,L=evolve(L,n);for(i=1,#L,print1(L[i],","));print())
```

The purpose of this document is to provide context, share underlying ideas and aesthetics.

Underlying canvas

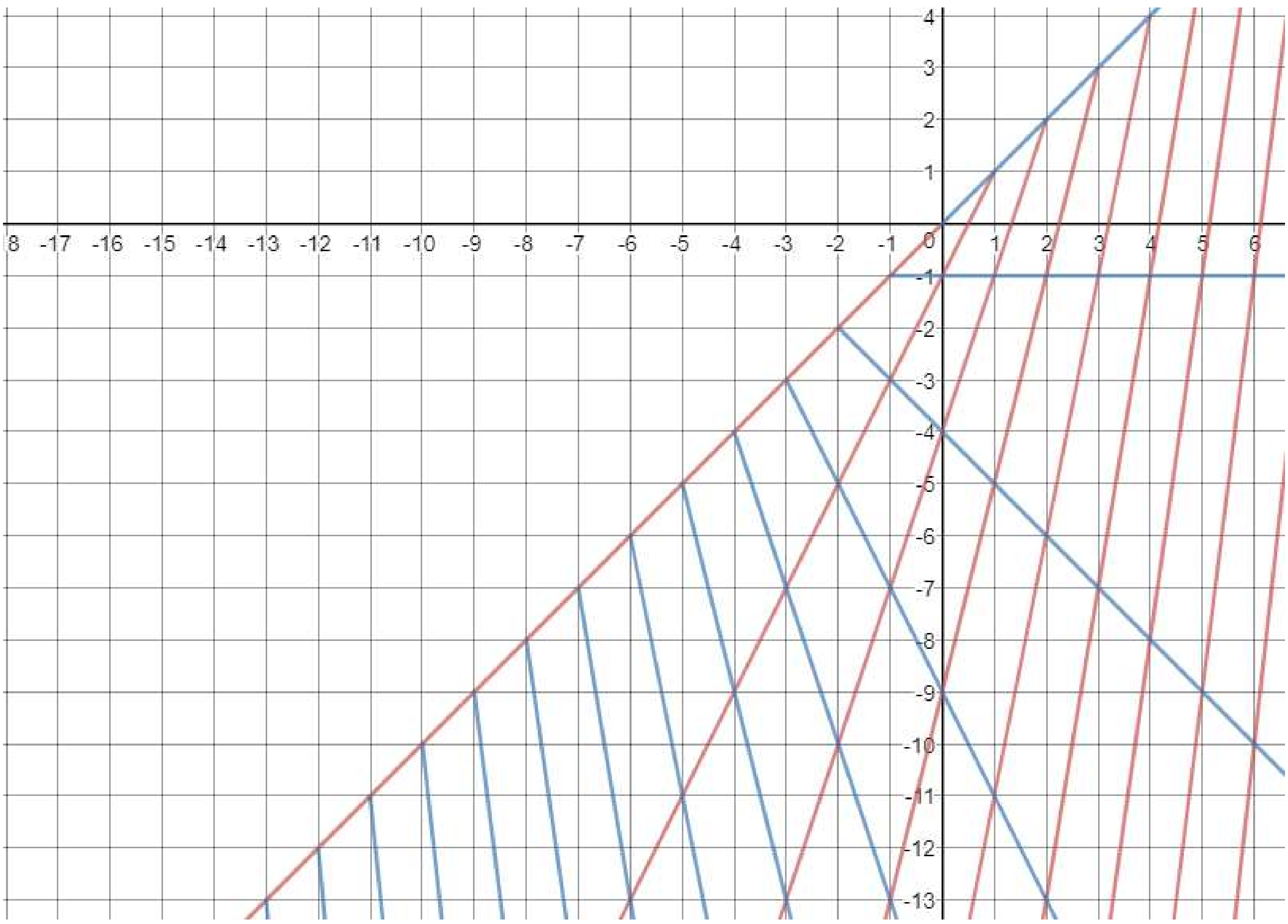
One starting point of thinking is sequence A293578, where the following triangle array made its appearance :

```
                1
              2 . 2
            3 . . . 3
          4 . . 3 . . 4
        5 . . . . . . . 5
      6 . . . 4 . 4 . . . 6
    7 . . . . . . . . . . . 7
  8 . . . . 5 . . . 5 . . . . 8
9 . . . . . . . 5 . . . . . . . 9
```

etc.

[Figure 1]

Transposed to a graphic plot with drawn and prolonged lines, we get :



[Figure 2]

The shape of A293578 is still recognizable in this plot. For instance, value « 1 » in the array [Figure 1] corresponds to coordinates (-2,-2) in the plot, the leftmost « 2 » to (-3,-3), etc.

We'll let the reader check that the equations of lines are :

- red lines (m = 1, 2, …) :

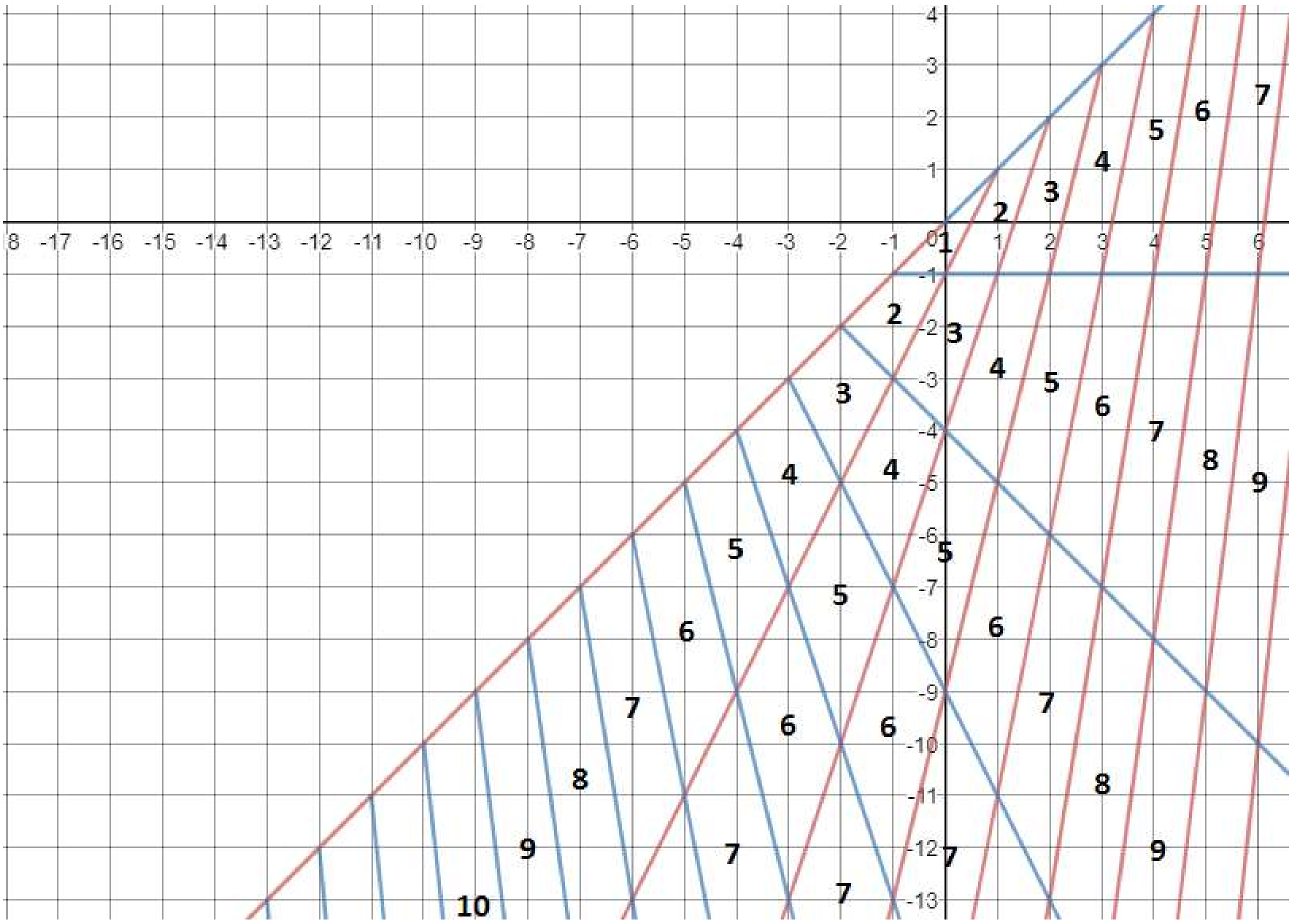
  $y = mx - (m - 1)^2$ for all $x <= m - 1$

- blue lines (n = -1, 0, …) :

  $y = -nx - (n + 1)^2$ for all $x >= 1 - n$

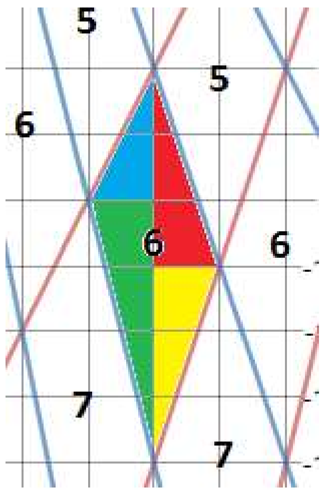
Anamorphosis with A002024

The plotted lines delimit quadrilaterals (except one degenerated case at the origin, which looks like a triangle instead). What is the area of each quadrilateral ? Graphical answer :



[Figure 3]

Visual proof :



decompose the quadrilateral in 4 right triangles ;
here, (1*2)/2 + (1*3)/2 + (1*4)/2 + (1*3)/2 = 6

[Figure 4]

For quadrilaterals in the upper-right region, the decomposition in right triangles may imply differences of areas instead of sums, but the result still holds that the area of each quadrilateral is a positive integer. Moreover, there is a global structure.

Look attentively at Figure 3. Haven't we just drawn an anamorphosis of

1	2	3	4	5	6	7	...
2	3	4	5	6	7	8	...
3	4	5	6	7	8	9	...
4	5	6	7	8	9	10	...
5	6	7	8	9	10	11	...
6	7	8	9	10	11	12	...
7	8	9	10	11	12	13	...
...	...	...	...	...	...	...	...

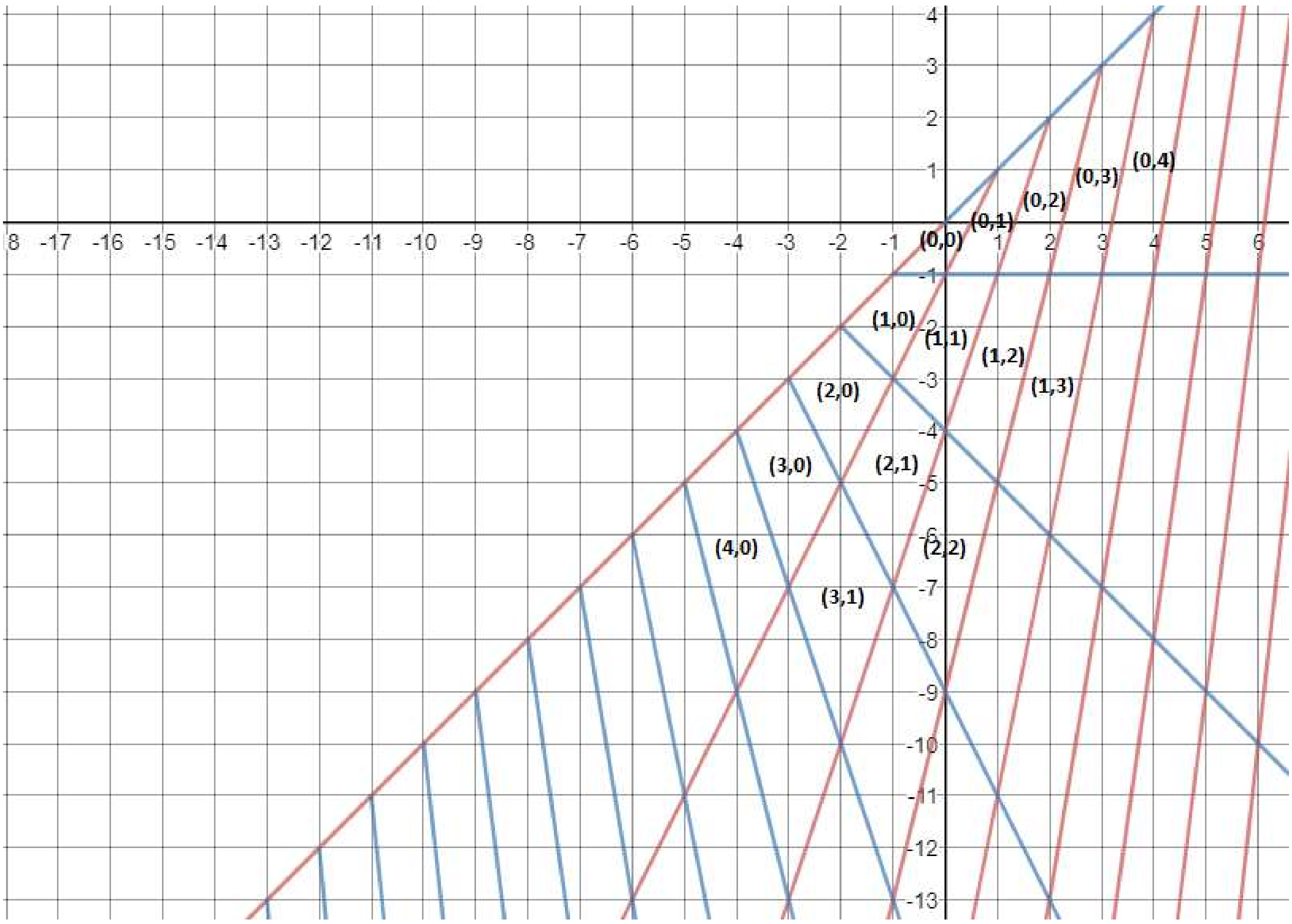
[Figure 5]

?

This array [Figure 5] is referred to as A002024 in the OEIS. Let's use the variable i (i = 0, 1, …) to index rows and the variable j (j = 0, 1, …) to index columns. We have:

$$A002024[i, j] = i + j + 1.$$

We can label each quadrilateral with the same kind of (i, j) :



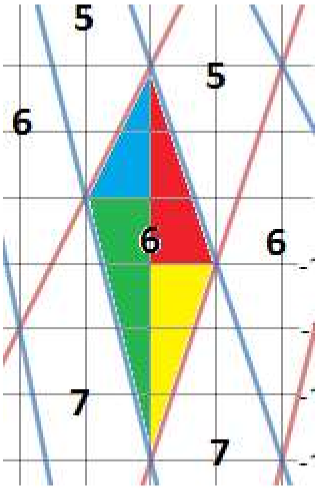
[Figure 6]

If Q[i, j] denotes the quadrilateral with an (i, j) label, then, from our previous observations,

$$\text{Area}(Q[i, j]) = A002024[i, j]$$

Numeric structure

Figure 4, hereafter recalled, shows that there is a numeric structure in each Q[i, j]. The following 6 is the one with label (i, j) = (3, 1) :

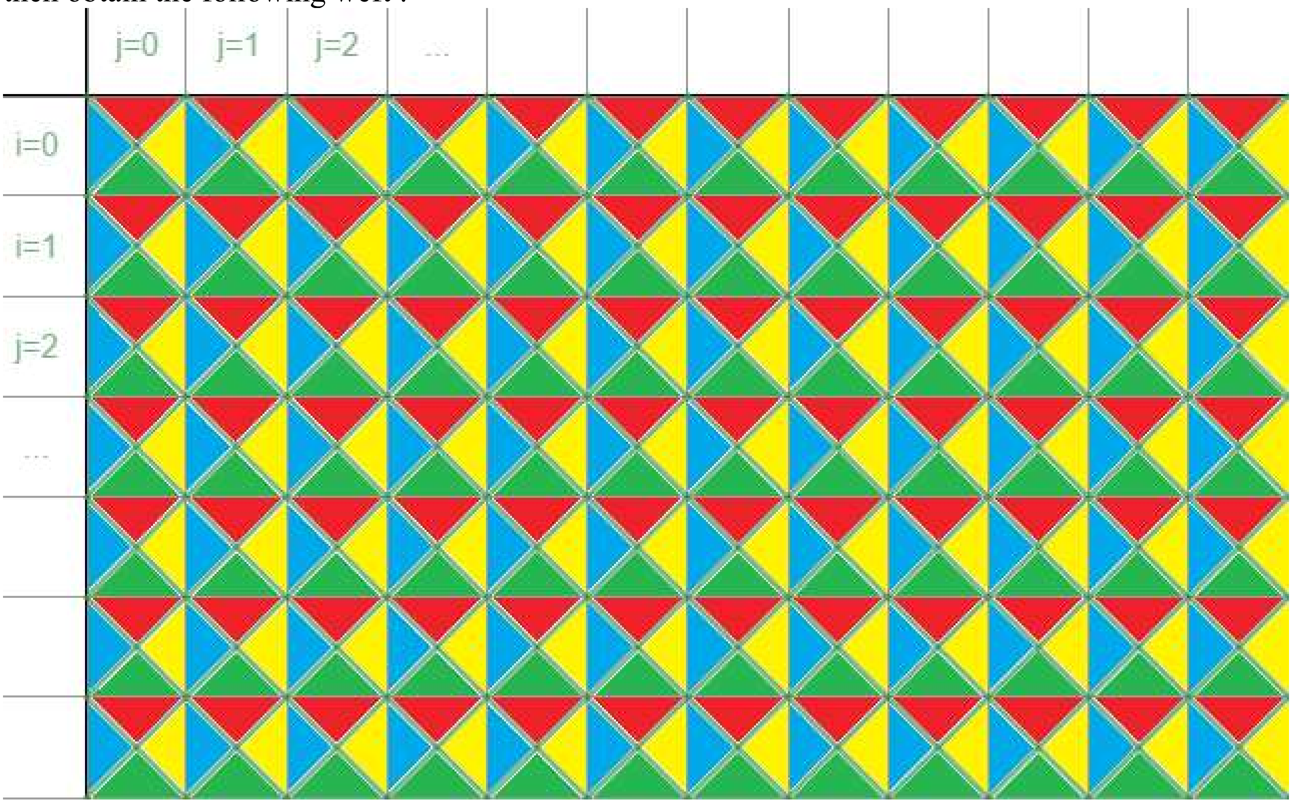


[Figure 4]

Each quadrilateral contains four numbers : NW(blue), NE(red), SW(green), SE(yellow), that we will define as the height[1] of each corresponding right triangle. Thus, in this example,

NW(3,1) = 2 ; NE(3,1) = 3 ; SW(3,1) = 4 ; SE(3,1) = 3.

The anamorphosis having a slanting effect, we may decide to straighten this colored structure. We then obtain the following weft :



[Figure7]

It looks wise to rename directions :

N = red ; W = blue ; S = green ; E=yellow

1 (a height that may have a minus sign in the upper-right region, for red values)

To represent numeric values in this straightened colored structure, we'll use the following text-pattern convention:

	N			N			N		…
W		E	W		E	W		E	…
	S			S			S		…
	N			N			N		…
W		E	W		E	W		E	…
	S			S			S		…
	N			N			N		…
W		E	W		E	W		E	…
	S			S			S		…
…	…	…	…	…	…	…	…	…	...

Time has come to fill this structure with the values of each Q[i, j] :

	-1			-1			-1		…
1		2	2		3	3		4	…
	0			0			0		…
	0			0			0		…
1		2	2		3	3		4	…
	1			1			1		…
	1			1			1		…
1		2	2		3	3		4	…
	2			2			2		…
…	…	…	…	…	…	…	…	…	...

Formulae :

N(i, j) = i – 1
W(i, j) = j + 1
S(i, j) = N(i +1, j) = i
E(i, j) = W(i, j + 1) = j + 2

Now, slant your head on the left and read by antidiagonals :

	1	−1	
	0	2	
1	0	2	−1
1	2	0	3

...

This is how A293670 originally emerged.

Luc Rousseau, October 14th, 2017.