A288119

First 9 terms

a_0	$a_{0+0} \neq 2*a_0$, so $a_0 \neq 0$. Hence, $\mathbf{a_0 = 1}$. Note that $a_{n+0} \neq a_n + a_0$ for any $n \geq 0$.
a_1	$a_1 \notin \{ a_0=1 \}$. Hence, $\mathbf{a_1 = 0}$.
a_2	$a_2 \notin \{ a_0=1, a_1=0, a_1+a_1=0 \}$. Hence, $\mathbf{a_2 = 2}$.
a_3	$a_3 \notin \{ a_0=1, a_1=0, a_2=2, a_1+a_2=2, a_2+a_1=2 \}$. Hence, $\mathbf{a_3 = 3}$.
a_4	$a_4 \notin \{ a_0=1, a_1=0, a_2=2, a_3=3, a_1+a_3=3, a_2+a_2=4, a_3+a_1=3 \}$. Hence, $\mathbf{a_4 = 5}$.
a_5	$a_5 \notin \{ a_0=1, a_1=0, a_2=2, a_3=3, a_4=5, a_1+a_4=5, a_2+a_3=5, a_3+a_2=5, a_4+a_1=5 \}$. Hence, $\mathbf{a_5 = 4}$.
a_6	$a_6 \notin \{ a_0=1, a_1=0, a_2=2, a_3=3, a_4=5, a_5=4, a_1+a_5=4, a_2+a_4=7, a_3+a_3=6, a_4+a_2=7, a_5+a_1=4 \}$. Hence, $\mathbf{a_6 = 8}$.
a_7	$a_7 \notin \{ a_0=1, a_1=0, a_2=2, a_3=3, a_4=5, a_5=4, a_6=8, a_1+a_6=8, a_2+a_5=6, a_3+a_4=8, a_4+a_3=8, a_5+a_2=6, a_6+a_1=8 \}$. Hence, $\mathbf{a_7 = 7}$.
a_8	$a_8 \notin \{ a_0=1, a_1=0, a_2=2, a_3=3, a_4=5, a_5=4, a_6=8, a_7=7, a_1+a_7=7, a_2+a_6=10, a_3+a_5=7, a_4+a_4=10, a_5+a_3=7,$ $a_6+a_2=10, a_7+a_1=7 \}$. Hence, $\mathbf{a_8 = 6}$.

$a_{9*k+i} = 9*k + a_i$ for any $k \geq 0$ and $0 \leq i \leq 8$

We show by induction that for any $k \geq 0$:
The sequence, restricted to 0…9*k+8, is bijective,
$a_{9*k+i} = 9*k + a_i$ for any i in 0…8.

The hypothesis is true for k=0:

- $a_0 = 1$
- $a_1 = 0$
- $a_2 = 2$
- $a_3 = 3$
- $a_4 = 5$
- $a_5 = 4$
- $a_6 = 8$
- $a_7 = 7$
- $a_8 = 6$

Suppose the hypothesis is true for some k; we show it for k+1:

$a_{9*(k+1)+0}$	$a_{9*(k+1)+0}$ must be distinct from: • a(0)…a(9*k+8) = 0…9*k+8 • $a_0 + a_{9*k+9} = 1 + a_{9*k+9}$ • $a_1 + a_{9*k+8} = 0 + 9*k + 6 = 9*k + 9$ • $a_2 + a_{9*k+7} = 2 + 9*k + 7 = 9*k + 13$ • $a_3 + a_{9*k+6} = 3 + 9*k + 8 = 9*k + 15$ • $a_4 + a_{9*k+5} = 5 + 9*k + 4 = 9*k + 19$ • $a_5 + a_{9*k+4} = 4 + 9*k + 5 = 9*k + 17$ • $a_6 + a_{9*k+3} = 8 + 9*k + 3 = 9*k + 25$ • $a_7 + a_{9*k+2} = 7 + 9*k + 2 = 9*k + 23$ • $a_8 + a_{9*k+1} = 6 + 9*k = 9*k + 21$ • There are no other contraint as $a_i + a_{9*(k+1)+0-i} = a_{i+9*n} + a_{9*(k+1-n)+0}$ for any n in 0…k and i in 0…8.

	Hence, **$a_{9*(k+1)+0} = 9*k + 10 = 9*(k+1) + a_0$**
$a_{9*(k+1)+1}$	$a_{9*(k+1)+1}$ must be distinct from: • a(0)…a(9*k+8) = 0…9*k+8 • a_{9*k+9}=9*k + 10 • $a_0 + a_{9*k+10} = 1 + a_{9*k+10}$ • $a_1 + a_{9*k+9}$ = 0 + 9*k + 10 = 9*k + 11 • $a_2 + a_{9*k+8}$ = 2 + 9*k + 6 = 9*k + 14 • $a_3 + a_{9*k+7}$ = 3 + 9*k + 7 = 9*k + 17 • $a_4 + a_{9*k+6}$ = 5 + 9*k + 8 = 9*k + 18 • $a_5 + a_{9*k+5}$ = 4 + 9*k + 4 = 9*k + 21 • $a_6 + a_{9*k+4}$ = 8 + 9*k + 5 = 9*k + 24 • $a_7 + a_{9*k+3}$ = 7 + 9*k + 3 = 9*k + 22 • $a_8 + a_{9*k+2}$ = 6 + 9*k + 2 = 9*k + 25 • There are no other contraint as $a_i + a_{9*(k+1)+1-i} = a_{i+9*n} + a_{9*(k+1-n)+1}$ for any n in 0…k and i in 0…8. Hence, **$a_{9*(k+1)+1} = 9*k + 9 = 9*(k+1) + a_1$**
$a_{9*(k+1)+2}$	$a_{9*(k+1)+2}$ must be distinct from: • a(0)…a(9*k+8) = 0…9*k+8 • a_{9*k+9}=9*k + 10 • a_{9*k+10}=9*k + 9 • $a_0 + a_{9*k+11} = 1 + a_{9*k+11}$ • $a_1 + a_{9*k+10}$ = 0 + 9*k + 9 = 9*k + 12 • $a_2 + a_{9*k+9}$ = 2 + 9*k + 10 = 9*k + 16 • $a_3 + a_{9*k+8}$ = 3 + 9*k + 6 = 9*k + 16 • $a_4 + a_{9*k+7}$ = 5 + 9*k + 7 = 9*k + 22 • $a_5 + a_{9*k+6}$ = 4 + 9*k + 8 = 9*k + 20 • $a_6 + a_{9*k+5}$ = 8 + 9*k + 4 = 9*k + 23 • $a_7 + a_{9*k+4}$ = 7 + 9*k + 5 = 9*k + 26 • $a_8 + a_{9*k+3}$ = 6 + 9*k + 3 = 9*k + 24 • There are no other contraint as $a_i + a_{9*(k+1)+2-i} = a_{i+9*n} + a_{9*(k+1-n)+2}$ for any n in 0…k and i in 0…8. Hence, **$a_{9*(k+1)+2} = 9*k + 11 = 9*(k+1) + a_2$**
$a_{9*(k+1)+3}$	$a_{9*(k+1)+3}$ must be distinct from: • a(0)…a(9*k+8) = 0…9*k+8 • a_{9*k+9}=9*k + 10 • a_{9*k+10}=9*k + 9 • a_{9*k+11}=9*k + 11 • $a_0 + a_{9*k+12} = 1 + a_{9*k+12}$ • $a_1 + a_{9*k+11}$ = 0 + 9*k + 11 = 9*k + 14 • $a_2 + a_{9*k+10}$ = 2 + 9*k + 9 = 9*k + 15 • $a_3 + a_{9*k+9}$ = 3 + 9*k + 10 = 9*k + 20 • $a_4 + a_{9*k+8}$ = 5 + 9*k + 6 = 9*k + 21 • $a_5 + a_{9*k+7}$ = 4 + 9*k + 7 = 9*k + 19 • $a_6 + a_{9*k+6}$ = 8 + 9*k + 8 = 9*k + 27 • $a_7 + a_{9*k+5}$ = 7 + 9*k + 4 = 9*k + 25 • $a_8 + a_{9*k+4}$ = 6 + 9*k + 5 = 9*k + 26 • There are no other contraint as $a_i + a_{9*(k+1)+3-i} = a_{i+9*n} + a_{9*(k+1-n)+3}$ for any n in 0…k and i in 0…8. Hence, **$a_{9*(k+1)+3} = 9*k + 12 = 9*(k+1) + a_3$**
$a_{9*(k+1)+4}$	$a_{9*(k+1)+4}$ must be distinct from: • a(0)…a(9*k+8) = 0…9*k+8 • a_{9*k+9}=9*k + 10 • a_{9*k+10}=9*k + 9 • a_{9*k+11}=9*k + 11 • a_{9*k+12}=9*k + 12 • $a_0 + a_{9*k+13} = 1 + a_{9*k+13}$ • $a_1 + a_{9*k+12}$ = 0 + 9*k + 12 = 9*k + 13 • $a_2 + a_{9*k+11}$ = 2 + 9*k + 11 = 9*k + 19 • $a_3 + a_{9*k+10}$ = 3 + 9*k + 9 = 9*k + 19 • $a_4 + a_{9*k+9}$ = 5 + 9*k + 10 = 9*k + 20 • $a_5 + a_{9*k+8}$ = 4 + 9*k + 6 = 9*k + 23 • $a_6 + a_{9*k+7}$ = 8 + 9*k + 7 = 9*k + 26 • $a_7 + a_{9*k+6}$ = 7 + 9*k + 8 = 9*k + 27

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$a_8 + a_{9*k + 5} = 6 + 9*k + 4 = 9*k + 27$
There are no other contraint as $a_i + a_{9*(k+1)+4-i} = a_{i+9*n} + a_{9*(k+1-n) + 4}$ for any n in 0…k and i in 0…8.

Hence, $a_{9*(k+1)+4} = 9*k + 14 = 9*(k+1) + a_4$
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<td>$a_{9*(k+1)+5}$</td>
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$a_{9*(k+1)+5}$ must be distinct from:

a(0)…a(9*k+8) = 0…9*k+8
$a_{9*k + 9}=9*k + 10$
$a_{9*k + 10}=9*k + 9$
$a_{9*k + 11}=9*k + 11$
$a_{9*k + 12}=9*k + 12$
$a_{9*k + 13}=9*k + 14$
$a_0 + a_{9*k + 14} = 1 + a_{9*k + 14}$
$a_1 + a_{9*k + 13} = 0 + 9*k + 14 = 9*k + 17$
$a_2 + a_{9*k + 12} = 2 + 9*k + 12 = 9*k + 18$
$a_3 + a_{9*k + 11} = 3 + 9*k + 11 = 9*k + 18$
$a_4 + a_{9*k + 10} = 5 + 9*k + 9 = 9*k + 24$
$a_5 + a_{9*k + 9} = 4 + 9*k + 10 = 9*k + 22$
$a_6 + a_{9*k + 8} = 8 + 9*k + 6 = 9*k + 28$
$a_7 + a_{9*k + 7} = 7 + 9*k + 7 = 9*k + 28$
$a_8 + a_{9*k + 6} = 6 + 9*k + 8 = 9*k + 29$
There are no other contraint as $a_i + a_{9*(k+1)+5-i} = a_{i+9*n} + a_{9*(k+1-n) + 5}$ for any n in 0…k and i in 0…8.

Hence, $a_{9*(k+1)+5} = 9*k + 13 = 9*(k+1) + a_5$
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<td>$a_{9*(k+1)+6}$</td>
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$a_{9*(k+1)+6}$ must be distinct from:

a(0)…a(9*k+8) = 0…9*k+8
$a_{9*k + 9}=9*k + 10$
$a_{9*k + 10}=9*k + 9$
$a_{9*k + 11}=9*k + 11$
$a_{9*k + 12}=9*k + 12$
$a_{9*k + 13}=9*k + 14$
$a_{9*k + 14}=9*k + 13$
$a_0 + a_{9*k + 15} = 1 + a_{9*k + 15}$
$a_1 + a_{9*k + 14} = 0 + 9*k + 13 = 9*k + 16$
$a_2 + a_{9*k + 13} = 2 + 9*k + 14 = 9*k + 17$
$a_3 + a_{9*k + 12} = 3 + 9*k + 12 = 9*k + 22$
$a_4 + a_{9*k + 11} = 5 + 9*k + 11 = 9*k + 23$
$a_5 + a_{9*k + 10} = 4 + 9*k + 9 = 9*k + 24$
$a_6 + a_{9*k + 9} = 8 + 9*k + 10 = 9*k + 29$
$a_7 + a_{9*k + 8} = 7 + 9*k + 6 = 9*k + 30$
$a_8 + a_{9*k + 7} = 6 + 9*k + 7 = 9*k + 28$
There are no other contraint as $a_i + a_{9*(k+1)+6-i} = a_{i+9*n} + a_{9*(k+1-n) + 6}$ for any n in 0…k and i in 0…8.

Hence, $a_{9*(k+1)+6} = 9*k + 17 = 9*(k+1) + a_6$
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<td>$a_{9*(k+1)+7}$</td>
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$a_{9*(k+1)+7}$ must be distinct from:

a(0)…a(9*k+8) = 0…9*k+8
$a_{9*k + 9}=9*k + 10$
$a_{9*k + 10}=9*k + 9$
$a_{9*k + 11}=9*k + 11$
$a_{9*k + 12}=9*k + 12$
$a_{9*k + 13}=9*k + 14$
$a_{9*k + 14}=9*k + 13$
$a_{9*k + 15}=9*k + 17$
$a_0 + a_{9*k + 16} = 1 + a_{9*k + 16}$
$a_1 + a_{9*k + 15} = 0 + 9*k + 17 = 9*k + 15$
$a_2 + a_{9*k + 14} = 2 + 9*k + 13 = 9*k + 21$
$a_3 + a_{9*k + 13} = 3 + 9*k + 14 = 9*k + 21$
$a_4 + a_{9*k + 12} = 5 + 9*k + 12 = 9*k + 25$
$a_5 + a_{9*k + 11} = 4 + 9*k + 11 = 9*k + 25$
$a_6 + a_{9*k + 10} = 8 + 9*k + 9 = 9*k + 31$
$a_7 + a_{9*k + 9} = 7 + 9*k + 10 = 9*k + 29$
$a_8 + a_{9*k + 8} = 6 + 9*k + 6 = 9*k + 32$
There are no other contraint as $a_i + a_{9*(k+1)+7-i} = a_{i+9*n} + a_{9*(k+1-n) + 7}$ for any n in 0…k and i in 0…8.

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	Hence, **$a_{9*(k+1)+7}$ = 9*k + 16 = 9*(k+1) + a_7**
$a_{9*(k+1)+8}$	$a_{9*(k+1)+8}$ must be distinct from: • a(0)…a(9*k+8) = 0…9*k+8 • a_{9*k+9}=9*k + 10 • a_{9*k+10}=9*k + 9 • a_{9*k+11}=9*k + 11 • a_{9*k+12}=9*k + 12 • a_{9*k+13}=9*k + 14 • a_{9*k+14}=9*k + 13 • a_{9*k+15}=9*k + 17 • a_{9*k+16}=9*k + 16 • a_0 + a_{9*k+17} = 1 + a_{9*k+17} • a_1 + a_{9*k+16} = 0 + 9*k + 16 = 9*k + 19 • a_2 + a_{9*k+15} = 2 + 9*k + 17 = 9*k + 20 • a_3 + a_{9*k+14} = 3 + 9*k + 13 = 9*k + 23 • a_4 + a_{9*k+13} = 5 + 9*k + 14 = 9*k + 26 • a_5 + a_{9*k+12} = 4 + 9*k + 12 = 9*k + 27 • a_6 + a_{9*k+11} = 8 + 9*k + 11 = 9*k + 30 • a_7 + a_{9*k+10} = 7 + 9*k + 9 = 9*k + 33 • a_8 + a_{9*k+9} = 6 + 9*k + 10 = 9*k + 31 • There are no other contraint as a_i + $a_{9*(k+1)+8-i}$ = a_{i+9*n} + $a_{9*(k+1-n)+8}$ for any n in 0…k and i in 0…8. Hence, **$a_{9*(k+1)+8}$ = 9*k + 15 = 9*(k+1) + a_8**

QED

The sequence is self-inverse

For any k ≥ 0:

- $a_{a_{9*k+0}}$ = a_{9*k+a_0} = 9*k + a_{a_0} = 9*k + a_1 = 9*k + 0
- $a_{a_{9*k+1}}$ = a_{9*k+a_1} = 9*k + a_{a_1} = 9*k + a_0 = 9*k + 1
- $a_{a_{9*k+2}}$ = a_{9*k+a_2} = 9*k + a_{a_2} = 9*k + a_2 = 9*k + 2
- $a_{a_{9*k+3}}$ = a_{9*k+a_3} = 9*k + a_{a_3} = 9*k + a_3 = 9*k + 3
- $a_{a_{9*k+4}}$ = a_{9*k+a_4} = 9*k + a_{a_4} = 9*k + a_5 = 9*k + 4
- $a_{a_{9*k+5}}$ = a_{9*k+a_5} = 9*k + a_{a_5} = 9*k + a_4 = 9*k + 5
- $a_{a_{9*k+6}}$ = a_{9*k+a_6} = 9*k + a_{a_6} = 9*k + a_8 = 9*k + 6
- $a_{a_{9*k+7}}$ = a_{9*k+a_7} = 9*k + a_{a_7} = 9*k + a_7 = 9*k + 7
- $a_{a_{9*k+8}}$ = a_{9*k+a_8} = 9*k + a_{a_8} = 9*k + a_6 = 9*k + 8