RELATIVE FREQUENCIES OF MULTIPLES OF ULAM NUMBERS

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The motivation for this note is to get a first impression of the relative frequencies of multiples of Ulam numbers, sequence A002858 in OEIS. We will use two measures for this.

Let u be an Ulam number, and Let U be the set of the first 100,000 Ulam numbers (note $u_{100,000}$ = 1,351,223 is the 100,000th Ulam number). Then the first measure is to count how many Ulam numbers u satisfy $k*u \in U$. For example, for k=3, the largest Ulam number u such that $3*u \in U$ is u = 450,335, and the total of numbers u that satisfy this condition is 1,043 (see Table 1, column 2 below).

The second measure is to count for Ulam numbers u, u ≤ 100,000, how many of them satisfy the condition that k*u is also an Ulam number. For example, for k=3, u = 99,222 is the largest u ≤ 100,000 such that 3*u is also an Ulam number. The total of Ulam numbers u, u ≤ 100,000 that satisfy such condition is 236 (see Table 1, column 3).

Table 1 below shows the counts for both measures. Perhaps somewhat surprisingly, there are very few values for the multiples k*u for k=2 and k=5 under both measures. In fact, sequence A068791 in OEIS lists the first Ulam numbers u such that 2*u is also an Ulam number, and to get the 30th number in the sequence we have to go all the way to u = 4,867,024. Similarly, A287613 in OEIS lists Ulam numbers u such that 5*u is also an Ulam number. In contrast to these very scarce Ulam numbers with the property that k*u is also an Ulam number, the multiples k*u with such property, with very frequent values, appear for k = 4, 6, 3, and even for k = 9 and k = 7.

Table 1 also shows a simple computation of relative frequencies by dividing for each k = 2, ..., 32, the number of Ulam numbers u such that k*u is also Ulam by the total of the columns (the case k=1 was excluded; U(1,2) = an Ulam number as in A002858).

Now, which of Measure 1 or Measure is “better”?

It seems that Measure 2, since it doesn’t fix the range where we are counting (ie. up to the 100,000th Ulam number), but adjusts the range by multiplying k*u. In fact, for k = 32, we had to look up to 3,188,096 (the 236,003th Ulam number) to verify that for u = 99,628, 32*u = 3,188,096 is also an Ulam number.

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Data for very large Ulam numbers were kindly provided by Jud McCranie.

U(1,2)	Measure 1	Measure 2	Frequency 1	Frequency 2
1*U(1,2)	100.000	7.584		
2*U(1,2)	26	22	0,14%	0,13%
3*U(1,2)	1043	236	5,48%	1,39%
4*U(1,2)	3842	1122	20,18%	6,62%
5*U(1,2)	148	74	0,78%	0,44%
6*U(1,2)	1823	827	9,58%	4,88%
7*U(1,2)	1002	540	5,26%	3,19%
8*U(1,2)	804	484	4,22%	2,86%
9*U(1,2)	983	655	5,16%	3,87%
10*U(1,2)	699	532	3,67%	3,14%
11*U(1,2)	629	520	3,30%	3,07%
12*U(1,2)	692	605	3,64%	3,57%
13*U(1,2)	597	575	3,14%	3,39%
14*U(1,2)	525	553	2,76%	3,26%
15*U(1,2)	497	553	2,61%	3,26%
16*U(1,2)	460	548	2,42%	3,23%
17*U(1,2)	446	551	2,34%	3,25%
18*U(1,2)	464	604	2,44%	3,57%
19*U(1,2)	383	519	2,01%	3,06%
20*U(1,2)	387	564	2,03%	3,33%
21*U(1,2)	332	527	1,74%	3,11%
22*U(1,2)	341	549	1,79%	3,24%
23*U(1,2)	313	539	1,64%	3,18%
24*U(1,2)	313	553	1,64%	3,26%
25*U(1,2)	332	610	1,74%	3,60%
26*U(1,2)	336	596	1,76%	3,52%
27*U(1,2)	316	576	1,66%	3,40%
28*U(1,2)	312	631	1,64%	3,72%
29*U(1,2)	275	602	1,44%	3,55%
30*U(1,2)	269	564	1,41%	3,33%
31*U(1,2)	230	548	1,21%	3,23%
32*U(1,2)	218	561	1,15%	3,31%
TOTAL	19.037	16.940	100,00%	100,00%

TABLE 1: RELATIVE FREQUENCIES FOR MULTIPLES OF ULAM NUMBERS THAT ARE ALSO ULAM NUMBERS

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