Example Plot and Justification for the Formula in Conjecture #4

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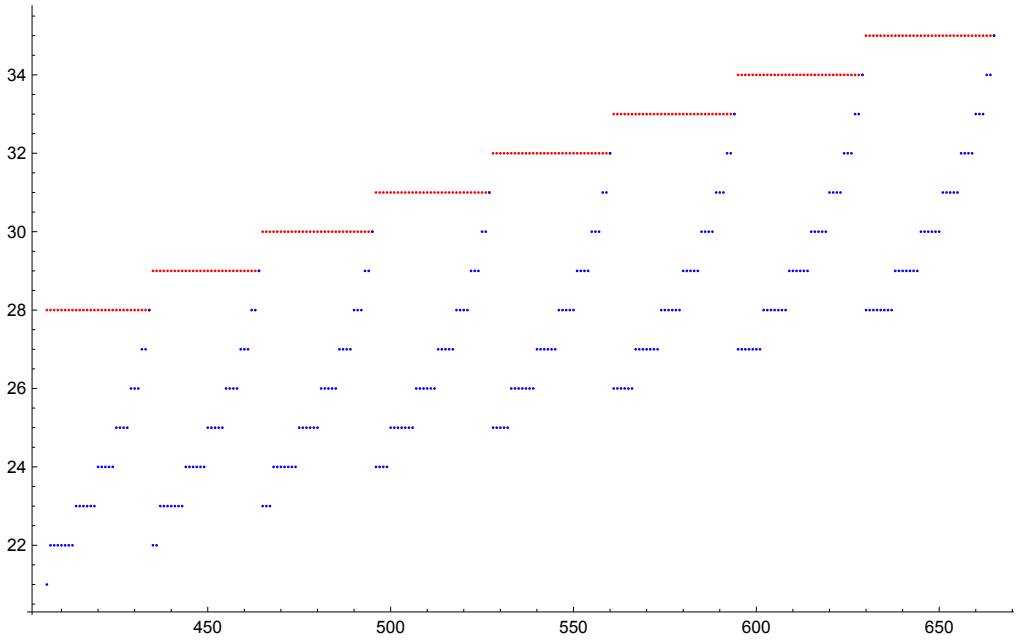
The Mathematica ListPlot[ ] below shows the column position of the rightmost inversion in a row of triangle A237591 starting at 406, the 7-th doubly triangular number, up to but not including 666, the 8-th doubly triangular number, as blue dots; the length of each row in the triangle is indicated by red dots. Note that the rightmost dot in a line of row lengths is also blue since the last entry in a row before a triangular number is 2, an inversion. Each of the eight "feathers" in the image contains numbers 1 ... 7 counting from the top down with the additional bottom row containing numbers 1 ... 8 increasing in each feather to the right.

```
ListPlot[{Map[{#, row[#]} &, Range[tr2[7], tr2[8] - 1]],
  Map[{#, trInversion[#]} &, Range[tr2[7], tr2[8] - 1]]},
 PlotStyle → {{Red, PointSize[0.0025]}, {Blue, PointSize[0.002]}}]
```



As an example for the computation of the position of the rightmost inversion we take n = 500 whose associated row in A237591 has length row(500) = 31. In order to find the correct level of the inversion in the "feather" we start counting down 0, 1, 2, ... from level 31. To this end we first compute the maximum number k with row(k) = 31 which is the number before the next triangular number from which we subtract 1: Binomial((row(500)+1)+1,2) - 1 = 527. Next we subtract 500 from that number and compute the largest triangular number less than or equal to that difference. That gives us the number of levels in the image that we need to descend, i.e., that we must subtract from row(500).

Generalizing this argument produces the function for the position of the rightmost inversion in row n of triangle A237591: A003056(n) - A003056(A000217(A003056(n) + 1) - 1 - n)
or as a Mathematica function

```
row[n_] := Floor[(Sqrt[8 n + 1] - 1) / 2]
trInversion[n_] := row[n] - row[Binomial[row[n] + 2, 2] - 1 - n]
```