Tendency of a(n)

The following plots show the relationship between the frequencies of d-triples and the sum of terms a(k):

Plot 1

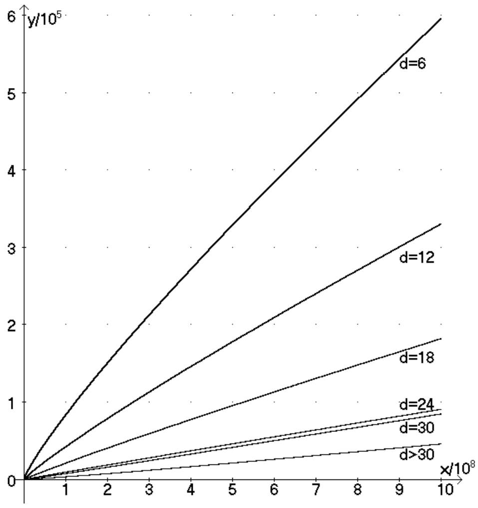
y = f(d,x): number of d-triples among the first x primes. Example: $f(6, 10^9) \approx 6\cdot 10^5$

Plot 2

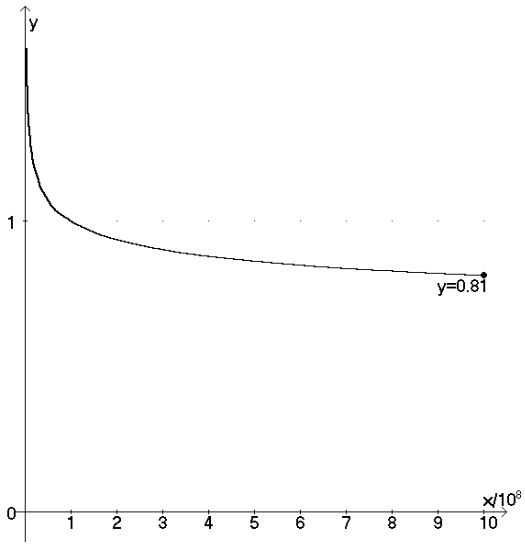
$$y(x) = \frac{f(6,x)}{\sum_{d>6} f(d,x)}$$ Example: $y(10^9) = 0.81$

Plot 3

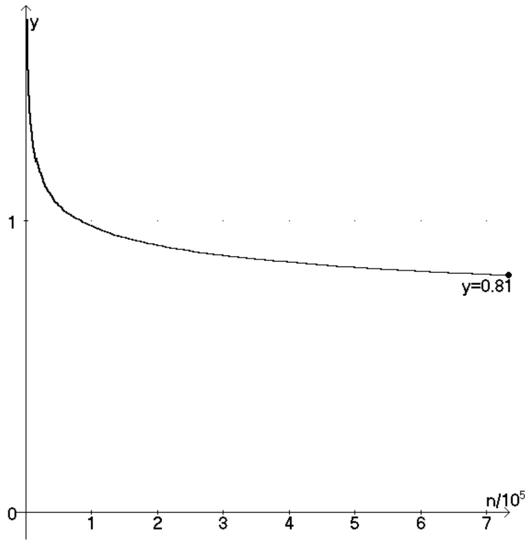
$$y(n) = \frac{\sum_{k=1}^{n} a(k)}{n}$$ Example: y(733000)=0.81



Plot 1



Plot 2



Plot 3

The curves in plot 2 and plot 3 look very similar, but the shapes are slightly different because the x/n-scales are not proportional to each other. Anyway, the y-values tend to the same limit, if it exists. My conjecture is that the limit is 0.

Let a generalized d-triple be the smallest of three primes p, p+d, p+2d. These are successive in “normal” d-triples. It is likely that the asymptotical frequencies of generalized d-triples do not depend on d. See link “Comparison of triples” in A279765.

Moreover, the asymptotical frequencies of normal and generalized d-triples should be the same because the average gap (difference) between consecutive primes asymptotically equals log(n) and so tends to infinity. For log(n)>>d, the probability that three primes p, p+d and p+2d are consecutive, is close to 1. So, for large n, there is no need to distinguish between normal and generalized d-triples. Thus a 6-triple is just one among many other equally frequent d-triples, and a(n) should tend to zero. The analyzed range of primes, however, is too small to reveal this tendency (see plots 2,3).

But even the analysis of a much wider range could not lead to a strict proof.