From Alec Jones, 30 March 2020

In what follows, I refer to the terms of A279212 in the form $T(n,k)$, referring to their row/column position in the table of A279212. I also use the convention of cardinal directions to refer to relative positions within this table, e.g. *all cells to the northwest of* $(5,3)$ means $(4,2)$ and $(3,1)$.

Theorem: For any column k in the table of A279212, there exists N_k so that, for any $n > N_k$, $T(n,k)$ is even. Thus, every column of A279212 contains a finite amount of odd terms.

Proof. We will proceed by strong induction, observing first that $T(n,1)$ is even for $n > 2$.

Let $K > 1$. Suppose that, for all $1 < k < K$, there is N_k so that, for any $n > N_k$, $T(n,k)$ is even. Let $N = \max_k N_k$, and consider the cell $(N+K-1,K)$. By the induction hypothesis, the cells to the west of this cell in the same row contain only even terms, and so their sum is even. Likewise, the cells to the northwest and southwest of $(N+K-1,K)$ in its diagonal and antidiagonal, respectively, contain only even terms, and so their sums are even. Denote by S the sum of the cells above $T(N+K-1,K)$ in column K; thus, whether or not S is even, we at least know that $T(N+K-1,K)$ is equal to S plus an even number.

Consider the cell $(N+K,K)$, which is directly beneath the cell we were just thinking about. As before, thanks to the induction hypothesis, the terms to its west, to its northwest, and to its southwest must all be even, and thus cannot contribute to odd parity for $T(N+K,K)$. Now consider the terms in the cells above $(N+K,K)$ in the same column. Their sum is $S+T(N+K-1,K)$; but since $T(N+K-1,K)$ is equal to S plus some even number, it follows that $T(N+K,K)$ is equal to $2S$ plus some even number, and so $T(N+K,K)$ is guaranteed to be even.

Let $M > 1$, and consider $T(N+K-1+M,K)$, which is an arbitrary cell beneath $(N+K,K)$. Consider that, for any cell beneath and including $(N+K,K)$ in column K, the cells to the west, to the northwest, and to the south do not contribute to odd parity, and may be neglected. Observe that, if $M=2$, we see that $T(N+K+1,K)$ is equal to $6S$ plus an even number. When $M=3$, we have $T(N+K+2,K)$ equal to $12S$ plus an even number. In general, we know that $T(N+K-1+M,K)$ is equal to $3S \cdot 2^{M-1}$ plus an even number, and so $T(N+K-1+M,K)$ is even. This completes the proof.