Rationals A274342 / A274343, Wolfdieter Lang, June 20, 2016

The rationals c(n), n >=2, defined in A274342 by their recurrence with inputs c(2) =c2 and c(3) = c3 are given here with increasing number of parts of the partitions of n with parts 2 and 3 only:

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n =  2:  c2,
n =  3:  c3,
n =  4:  (1/3)*c2^2,
n =  5:  (3/11)*c2*c3,
n =  6:  (1/13)*c3^2 + (2/39)*c2^3,
n =  7:  (2/33)*c2^2*c3,
n =  8:  (60/2431)*c2*c3^2 + (5/663)*c2^4,
n =  9:  (1/247)*c3^3 + (29/2717)*c2^3*c3,
n = 10:  (485/80223)*c2^2*c3^2 + (2/1989)*c2^5,
n = 11:  (1722/1062347)*c2*c3^3 + (5446/3187041)*c2^4*c3,
n = 12:  (3/16055)*c3^4 + (8000/6605027)*c2^3*c3^2 + (10/77571)*c2^6,
n = 13:  (5300/11685817)*c2^2*c3^3 + (270/1062347)*c2^5*c3,
n = 14:  (181188/2002524095)*c2*c3^4 + (955290/4405553009)*c2^4*c3^2 + (4/249951)*c2^7,
n = 15:  (4/497705)*c3^5 + (15988040/155409680283)*c2^3*c3^3 + (416012/11559397707)*c2^6*c3,
n = 16:  (32420068/1123416017295)*c2^2*c3^4 + (2682744/74894401153)*c2^5*c3^2 + (223/114727509)*c2^8,
n = 17:  (25851/5643476995)*c2*c3^5 + (8409205/409716429837)*c2^4*c3^3 + (49871/10158258591)*c2^7*c3,
n = 18:  (301/909705199)*c3^6 + (1713301109422/233400836858808047)*c2^3*c3^4
         + (1066033105795/1909643210662974 93)*c2^6*c3^2 + (4270/18394643943)*c2^9,
n = 19:  (57425882/34825896536145)*c2^2*c3^5 + (859704866/229850917138557)*c2^5*c3^3
         + (11125766/17096349208653)*c2^8*c3,
n = 20:  (77746116/357856262339147)*c2*c3^6 + (39343318862281/242916409438436375 07)*c2^4*c3^4
         + (501010332520/602272089516784401)*c2^7*c3^2 + (4762/174041631153)*c2^10.
. . .
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This can be compared this with the polynomials c(k) = c_k , for k = 4..19 of the Abramowitz - Stegun handbook, 18.5.9 – 18.5.24, where the partitions of n with parts 2 and 3 only are listed with descreasing number of parts.

The Eisenstein series with even indices are then given by G_{2*n} = c(n) / (2*n – 1), and with c(2) = g_2 / 20 and c(3) = g_3 / 28, they become, for n=2..20:

```
n =  2:   (1/60)*g2,
n =  3:   (1/140)*g3,
n =  4:   (1/8400)*g2^2,
n =  5:   (1/18480)*g2*g3,
n =  6:   (1/112112)*g3^2 + (1/1716000)*g2^3,
n =  7:   (1/2402400)*g2^2*g3,
n =  8:   (1/9529520)*g2*g3^2 +  (1/318240000)*g2^4,
n =  9:   (1/92176448)*g3^3 + (29/10346336000)*g2^3*g3,
n = 10:   (97/95600144640)*g2^2*g3^2 + (1/60465600000)*g2^5,
n = 11:   (41/233206413440)*g2*g3^3 + (389/21416915520000)*g2^4*g3,
n = 12:   (3/226970947840)*g3^4 + (1/119101846864)*g2^3*g3^2 + (1/11418451200000)*g2^6,
n = 13:   (53/25652705478400)*g2^2*g3^3 + (27/237965728000000)*g2^5*g3,
n = 14:   (719/2637564536006400)*g2*g3^4 + (4549/71052758929152000)*g2^4*g3^2 + (1/2159576640000000)*g2^7,
n = 15:   (1/62101224989440)*g3^5  (399701/19787009149120012800)*g2^3*g3^3
           + (104003/15017969500934400000 0)*g2^6*g3,
n = 16:   (8105017/21405946261320741120 00)*g2^2*g3^4 + (335343/72809341024900480 0000)*g2^5*g3^2
           + (223/9104775114240000000 0)*g2^8,
n = 17:   (1231/3052541356338073600)*g2*g3^5 + (240263/135682348451108659 2000)*g2^4*g3^3
           + (49871/12014375600747520000000)*g2^7*g3,
n = 18:    (43/2191890574482452480)*g3^6 + (122378650673/286922449560575037873 6640000)*g2^3*g3^4
           +(30458088737/95818257738225430087680 00000)*g2^6*g3^2 + (61/47090288494080000000 00)*g2^9,
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n = 19: (28712941/4435312065345657560064000)*g2^2*g3^5
+(429852433/298704690115115713228800000)*g2^5*g3^3
+(5562883/226711267586105702400000000)*g2^8*g3,
n = 20: (925549/1601297871149927775784960)*g2*g3^6
+ (5620474123183/13309922545901418042619453440000)*g2^4*g3^4
+ (12525258313/589282285090086395039232000000)*g2^7*g3^2
+ (2381/34752632908631040000000000)*g2^10.
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