

Extend

Math Student 12 (1944)

2752

(MST)

Hence

$$\begin{aligned} \sum PA^2 &= \frac{1}{4} \sum a^2 + \frac{1}{16} \sum a^2 + \frac{1}{64} \sum a^2 + \dots \text{to } \infty + 3PG^2 \\ &= \frac{1}{3} \sum a^2 + 3PG^2 \end{aligned}$$

But we know that

$$3 \sum a^2 = 4 (AA_1^2 + BB_1^2 + CC_1^2) = 9 \sum GA^2$$

$$\therefore \sum PA^2 = \sum GA^2 + 3PG^2$$

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4. Sequences whose k^{th} differences form a geometrical progression.

Let the k^{th} order differences of a sequence u_n be the geometric progression a, ar, ar^2, \dots . Then the succeeding order of differences also form geometrical progressions, whose first terms are

$$a(r-1), a(r-1)^2, a(r-1)^3, \dots$$

$$\begin{aligned} \text{Hence } u_{n+1} &= u_1 + {}_n C_1 \Delta u_1 + {}_n C_2 \Delta^2 u_1 + \dots + {}_n C_n \Delta^n u_1 \\ &= u_1 + {}_n C_1 \Delta u_1 + \dots + {}_n C_{k-1} \Delta^{k-1} u_1 + R_k \end{aligned}$$

$$\text{where } R_k = a \{ {}_n C_k + {}_n C_{k+1} (r-1) + \dots + {}_n C_n (r-1)^{n-k} \}$$

$$= \frac{a}{(r-1)^k} \left\{ (1+r-1)^n - 1 - {}_n C_1 (r-1) - \dots - {}_n C_{k-1} (r-1)^{k-1} \right\}$$

$$\begin{aligned} \therefore u_{n+1} &= \frac{ar^n}{(r-1)^k} + \left\{ u_1 - \frac{a}{(r-1)^k} \right\} + {}_n C_1 \left\{ \Delta u_1 - \frac{a}{(r-1)^{k-1}} \right\} + \dots \\ &\quad \dots + {}_n C_{k-1} \left\{ \Delta^{k-1} u_1 - \frac{a}{(r-1)} \right\} \end{aligned}$$

This method avoids the setting up of undetermined coefficients as in Hall and Knight's *Higher Algebra* p. 330 or working back from the order in G.P. as in Smith's *Algebra* p. 416.

Example: Find the n^{th} term of the series

	u_n :	1	2	4	10	29	88	268	812
Here	Δu_n		1	2	6	19	59	180	544
	$\Delta^2 u_n$			1	4	13	40	121	364
	$\Delta^3 u_n$				3	9	27	81	243

Thus $\Delta^3 u_n$ is a G.P. and $\Delta^2 u_1, \Delta^3 u_1, \dots$ are respectively 3, 3.2, 3.2², ...

$$\therefore u_{n+1} = \frac{3^{n+1}}{2^3} + \left[1 - \frac{3}{2^3} \right] + {}_n C_1 \left[1 - \frac{3}{2^2} \right] + {}_n C_2 \left[1 - \frac{3}{2} \right]$$

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2
4
10
29
88
268
812
2449
7366
22124
66406
199261
597836
1793572
5380792

2, 2752

16142465
48427498
145282612
435847970
1307544061
3922632352
1 1767897244

2632352 2752

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104      C 2752
105      CALL CARDA(2752,
106      DO 105 I=1,25
107      K=KO(I)
108      105  CALL POWR(3,I,NO(K)) ✓
109      K=KO(1)
110      CALL ADD1(NO(K),1,NO(K)) ✓
111      DO 106 I=2,25
112      K=KO(I-1)
113      KA=KO(I)
114      106  CALL ADD(NO(KA),NO(K),NO(KA)) ✓
115      K=KO(1)
116      CALL ADD1(NO(K),2,NO(K)) ✓
117      DO 107 I=2,25
118      K=KO(I-1)
119      KA=KO(I)
120      107  CALL ADD(NO(KA),NO(K),NO(KA))
121      K=KO(1)
122      CALL ADD1(NO(K),4,NO(K))
123      DO 108 I=2,25
124      K=KO(I-1)
125      KA=KO(I)
126      108  CALL ADD(NO(KA),NO(K),NO(KA))
127      CALL MZERO(NA,2)
128      CALL CARDB(NA)
129      CALL MZERO(NA,4)
130      CALL CARDB(NA)
131      DO 109 N=1,25
132      IF(NSW2,LT,0) GO TO 110
133      K=KO(N)
134      109  CALL CARDB(NO(K))
135      110  CONTINUE
136      CE 2752
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