Number of positions of a polyiamond tile in a triangle frame

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We need 5 parameters of the tile.

Parameter	Question	yes	no
a	Is the tile invariant on a vertical flip or invariant on a 180° rotation?	a = 0	a = 1
b	Is the tile invariant on a horizontal flip?	b = 1	b = 2
c	Is the tile invariant on a 120° rotation?	c = 1	c = 3

| Parameter	Value of Paramter (*see sample below*)
d	side length of the circumscribed upright triangle for the tile in standard orientation (you can define any orientation as standard orientation)
e	side length of the circumscribed upright triangle for the tile in vertically flipped standard orientation

Number N of positions in a triangle frame of order n (= side length n)

$$N(n) = b \cdot c \cdot T(n + 1 - d) + a \cdot b \cdot c \cdot T(n + 1 - e)$$

Where $T(n) = \tfrac{1}{2}n \cdot (n + 1)$ if $n \geq 0$ (triangular number) (*also works for* n = –1)
and $T(n) = 0$ if $n < 0$

Sample: sphinx tile

For a sphinx tile we have a = 1, b = 2, c = 3, d = 3, e = 5
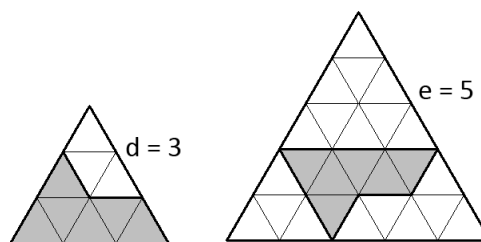
Thus $N(n) = 6\,T(n - 2) + 6\,T(n - 4)$ *works for all* n

$N(n) = 3 \cdot (n - 2) \cdot (n - 1) + 3 \cdot (n - 4) \cdot (n - 3)$ *only works for* n > 2
$N(n) = 6\,n^2 - 30\,n + 42$
$N(n) = 6\,(\,n^2 - 5\,n + 7\,)$