

Sums of the type

$$\sum_{i=0}^{\infty} \frac{(-1)^k}{(2i+1)^3}.$$

For  $k=0$  [A233091]:

$$\frac{7}{8} \zeta(3) = 1 + \frac{1}{3^3} + \frac{1}{5^3} + \frac{1}{7^3} + \frac{1}{9^3} + \frac{1}{11^3} + \frac{1}{13^3} + \frac{1}{15^3} + \frac{1}{17^3} + \dots$$

For  $k=i$ ,  $(-1)^k = 1, -1, 1, -1, 1, -1, \dots$  [A153071]:

$$\frac{\pi^3}{32} = 1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \frac{1}{9^3} - \frac{1}{11^3} + \frac{1}{13^3} - \frac{1}{15^3} + \frac{1}{17^3} - \dots$$

For  $k = \text{floor}(i:2)$ ,  $(-1)^k = 1, 1, -1, -1, 1, 1, \dots$  [A251809]:

$$\frac{3}{128} \sqrt{2} \pi^3 = 1 + \frac{1}{3^3} - \frac{1}{5^3} - \frac{1}{7^3} + \frac{1}{9^3} + \frac{1}{11^3} - \frac{1}{13^3} - \frac{1}{15^3} + \frac{1}{17^3} + \dots$$

For  $k = \text{floor}(i:3)$ ,  $(-1)^k = 1, 1, 1, -1, -1, -1, 1, 1, 1, \dots$  [A251967]:

$$\frac{29}{864} \pi^3 = 1 + \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} - \frac{1}{9^3} - \frac{1}{11^3} + \frac{1}{13^3} + \frac{1}{15^3} + \frac{1}{17^3} - \dots$$

For  $k = \text{floor}(i:4)$ ,  $(-1)^k = 1, 1, 1, 1, -1, -1, -1, -1, 1, 1, 1, 1, -1, -1, -1, -1, \dots$ :

$$\frac{1}{512} \sqrt{274 + 17\sqrt{2}} \pi^3 = 1 + \frac{1}{3^3} + \frac{1}{5^3} + \frac{1}{7^3} - \frac{1}{9^3} - \frac{1}{11^3} - \frac{1}{13^3} - \frac{1}{15^3} + \frac{1}{17^3} + \dots$$

For  $k = \text{floor}(i:5)$ ,  $(-1)^k = 1, 1, 1, 1, 1, -1, -1, -1, -1, -1, 1, 1, 1, 1, 1, \dots$ :

$$\frac{(1 + 60\sqrt{5}) \pi^3}{4000} = 1 + \frac{1}{3^3} + \frac{1}{5^3} + \frac{1}{7^3} + \frac{1}{9^3} - \frac{1}{11^3} - \frac{1}{13^3} - \frac{1}{15^3} - \frac{1}{17^3} - \frac{1}{19^3} + \dots$$