A244041 Base 4/3 Sum of Digits

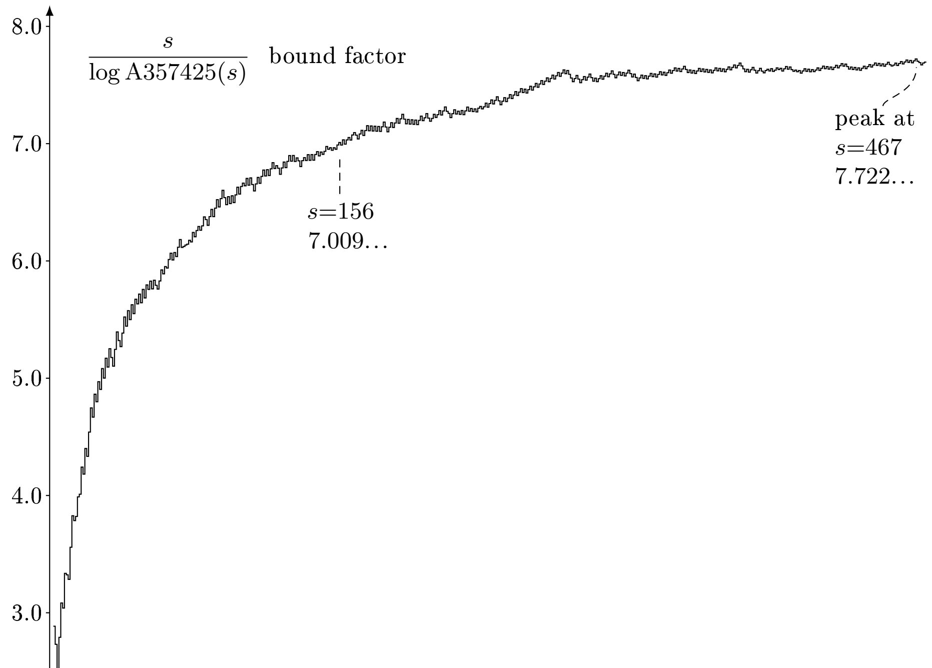
Kevin Ryde, March 2024

A244041 is the sum of digits of n in fractional base 4/3. Charles Greathouse in a comment gives the following bounds, and wonders whether factor 11 might be reduced to 7 or 8.

$$a(n) < 3\log_{4/3} n < 11 \log n \qquad \text{for } n \geq 2$$

$3\log_{4/3} n$ would be every digit 3, with the log as a proxy for the number of base 4/3 digits in n. The log exceeds that number since any $n \geq 3$ starts with a digit 3 so the smallest number with k digits is at least $n = 3.(\frac{4}{3})^{k-1}$ which has $\log_{4/3} n = k + 2.818\ldots$ Factor 11 is $3/\log(4/3) = 10.428\ldots$ rounded up.

One way to experiment with what factor might be needed is to take a sum of digits s and find the smallest n where sum $a(n){=}s$ occurs. This is $n{=}$A357425(s) and the following plot is the factor needed to cover s.



Prospective factor 7 is surpassed at $s = 156$ where the plot is still rising, though at a slowing rate.

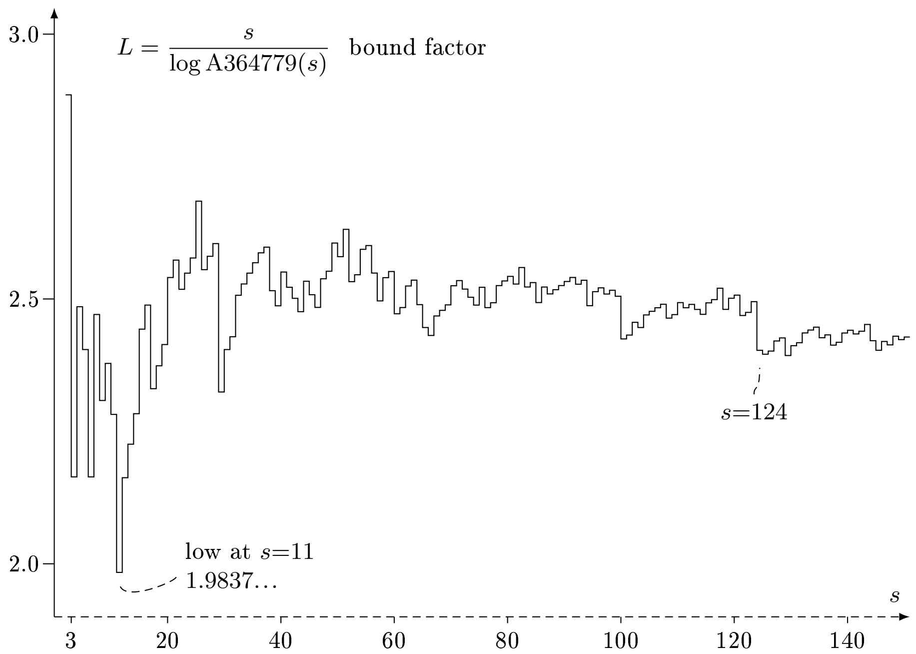
The peak so far is near the end $s{=}467$ with $n{=}183376210813834725647961191$ for $s/\log n = 7.722\ldots$ so a factor at least that large is required.

See A363758 for the largest sum s possible within a given number of digits.

A corresponding possible lower bound factor L would be,

$$L \log n < a(n) \qquad \text{for } n \geq 1$$

Again one way to experiment with such a factor is to take a sum of digits s and find the largest n where sum $a(n) = s$ occurs. This is $n = \text{A364779}(s)$ and the following plot is the factor L needed for a given s.



The low (so far) at s=11 might be an initial exception, but it's not particularly obvious whether the rest might be working its way down, or converging on something.

Drops such as at s=124 are where A364779(s) has a relatively large increase.

See A364751 for the smallest sum s possible within a given number of digits.