

The sum rhymes

The idea is to produce a ~~permutation of the natural numbers~~ thanks to a *Fibonacci-like* twist.

Let's start **T** with $a(1)=1$ and $a(2)=2$. We want now $a(n)$ to be the sum "s" of the last two digits of **T** - with a caveat: if "s" is already in **T**, we'll replace "s" with the smallest unused integer so far that ends with the same digit as "s" (the "rhyme" is there).

We get:

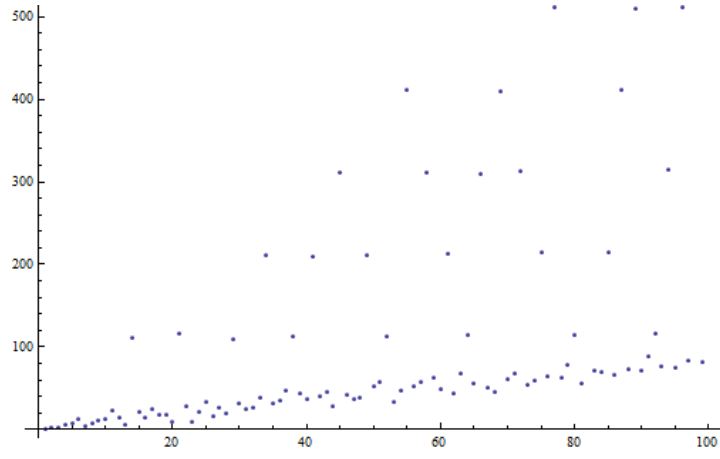
T = 1, 2, 3, 5, 8, 13, 4, 7, 11, 12, 23, 15, 6, 111, ...

Explanation:

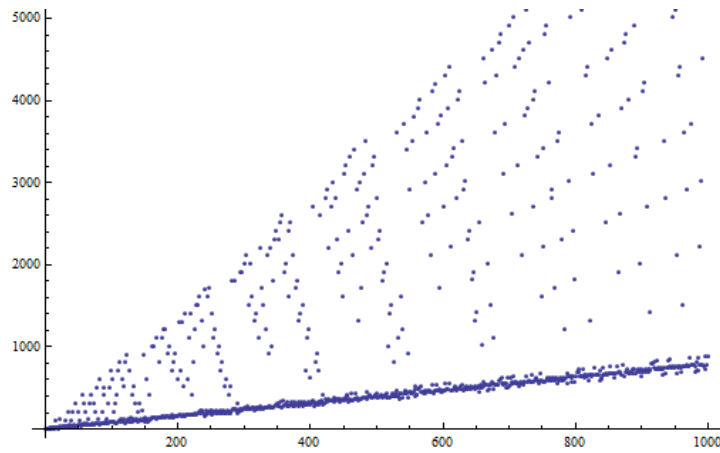
3 is 1+2
 5 is 2+3
 8 is 3+5
 13 is 5+8
 4 is 1+3
 7 is 3+4
 11 is 4+7
 12 should be 2 (1+1) but 2 is already in **T**; we replace 2 with 12 (because 12 ends with "2")
 23 should be 3 (1+2) but 3 (and 13) are already in **T**; we'll use 23 instead (because 23 ends with "3")
 15 should be 5 (2+3) but 5 is already in **T**; we replace 5 with 15 instead (because 15 ends with "5")
 6 is 1+5
 111 should be 11 (6+5) but ...

Here are the first 500 terms of **T**, computed by **Jim Nastos** - followed by 3 nice graphics computed by **Jean-Marc Falcoz**:

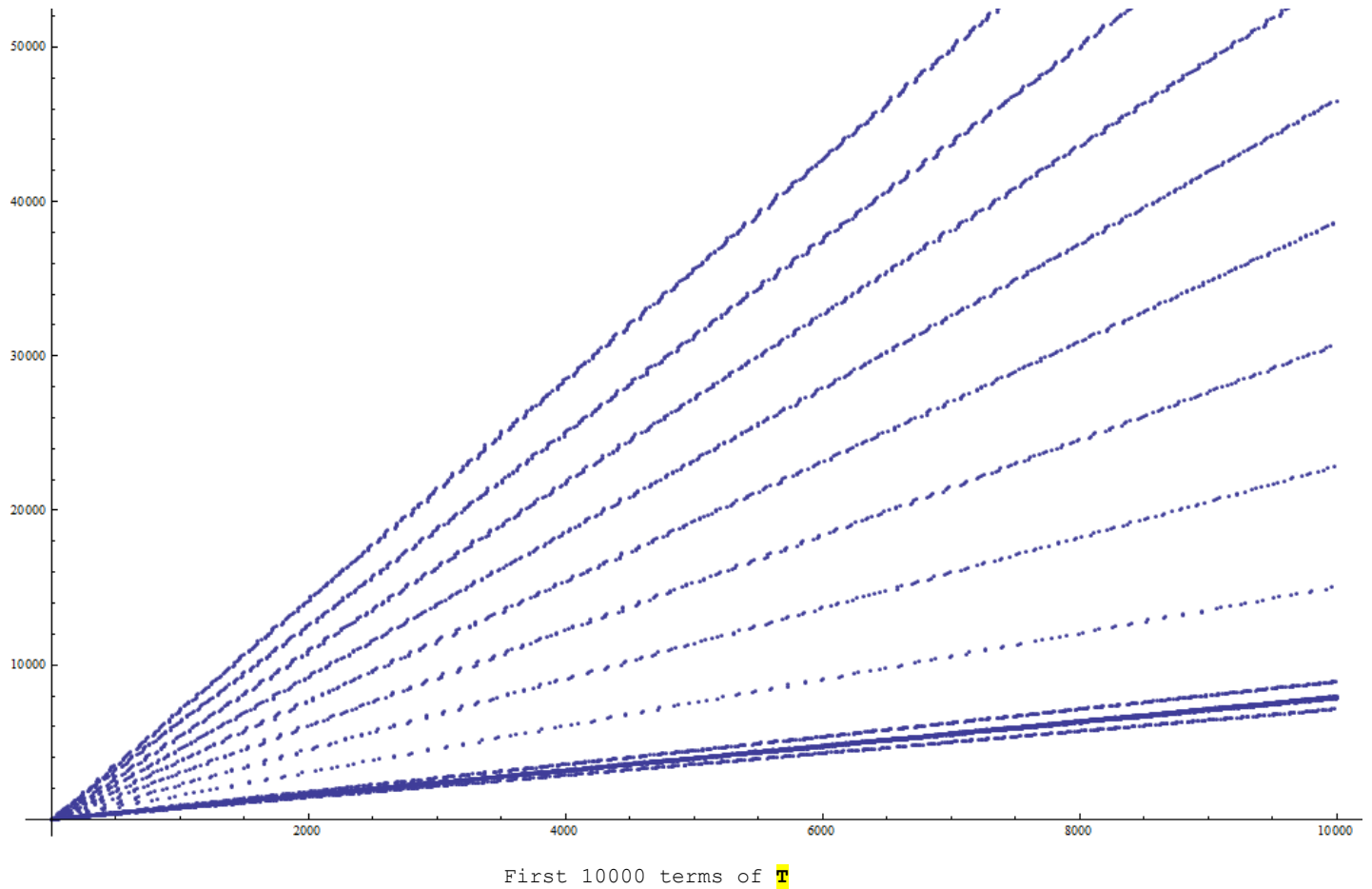
1, 2, 3, 5, 8, 13, 4, 7, 11, 12, 23, 15, 6, 111, 22, 14, 25, 17, 18, 9, 117, 28, 10, 21, 33, 16, 27, 19, 110, 31, 24, 26, 38, 211, 32, 35, 48, 112, 43, 37, 210, 41, 45, 29, 311, 42, 36, 39, 212, 53, 58, 113, 34, 47, 411, 52, 57, 312, 63, 49, 213, 44, 68, 114, 55, 310, 51, 46, 410, 61, 67, 313, 54, 59, 214, 65, 511, 62, 78, 115, 56, 611, 72, 69, 215, 66, 412, 73, 510, 71, 88, 116, 77, 314, 75, 512, 83, 711, 82, 610, 81, 79, 216, 87, 315, 76, 413, 64, 710, 91, 810, 101, 121, 93, 612, 103, 123, 85, 513, 74, 811, 92, 911, 102, 122, 84, 712, 133, 86, 414, 95, 514, 105, 125, 97, 316, 107, 127, 89, 217, 98, 317, 108, 118, 99, 218, 109, 119, 910, 131, 94, 613, 104, 124, 96, 415, 106, 126, 128, 1010, 141, 135, 138, 1011, 132, 145, 129, 1111, 142, 136, 139, 812, 143, 137, 1110, 151, 146, 1210, 161, 147, 1211, 152, 157, 912, 153, 148, 1012, 163, 149, 713, 134, 167, 813, 144, 158, 913, 154, 159, 614, 155, 1310, 171, 168, 714, 165, 1311, 162, 178, 515, 156, 1411, 172, 169, 615, 166, 1112, 173, 1410, 181, 179, 416, 177, 814, 175, 1212, 183, 1511, 182, 1510, 191, 1610, 201, 221, 193, 1312, 203, 223, 185, 1013, 164, 1710, 231, 174, 1611, 192, 1711, 202, 222, 184, 1412, 233, 176, 1113, 194, 1213, 204, 224, 186, 914, 195, 1014, 205, 225, 187, 715, 196, 815, 206, 226, 188, 516, 197, 616, 207, 227, 189, 417, 198, 517, 208, 228, 1810, 241, 235, 238, 1811, 232, 245, 199, 318, 209, 219, 1910, 251, 236, 229, 1911, 242, 246, 2010, 261, 237, 2110, 271, 248, 1512, 243, 247, 2011, 252, 257, 1612, 253, 258, 1313, 234, 267, 1413, 244, 268, 1114, 255, 2210, 281, 239, 1712, 263, 249, 1513, 254, 259, 1214, 265, 2111, 262, 278, 915, 256, 2211, 272, 269, 1015, 266, 1812, 273, 2310, 291, 2410, 301, 321, 283, 2311, 282, 2510, 331, 264, 2610, 341, 275, 1912, 293, 2012, 303, 323, 285, 1613, 274, 2411, 292, 2511, 302, 322, 284, 2112, 333, 276, 1713, 294, 1813, 304, 324, 286, 1314, 295, 1414, 305, 325, 277, 1514, 335, 288, 716, 287, 1115, 296, 1215, 306, 326, 298, 617, 308, 328, 2710, 351, 336, 279, 816, 297, 916, 307, 327, 289, 717, 338, 2611, 332, 345, 299, 418, 309, 319, 2810, 361, 337, 2910, 371, 348, 2212, 343, 347, 2711, 342, 346, 3010, 381, 329, 2811, 352, 357, 2312, 353, 358, 1913, 334, 367, 2013, 344, 368, 1614, 355, 3110, 391, 3210, 401, 421, 363, 339, 2412, 373, 3310, 431, 354, 349, 2113, 364, 3410, 441, 365, 2911, 362, 378, 1315, 356, 3011, 372, 359, 1714, 375, 2512, 383, 3111, 382, 3510, 451, 366, 2612, 393, 2712, 403, 423, 385, 2213, 374, 3211, 392, 3311, 402, 422, 384, 2812, ...



First 100 terms of **T**



First 1000 terms of **T**



[Jim Nastos]

Hi Eric,
 Nice sequence, but be careful... it is not a permutation of the Naturals. For instance, 100 (or any number ending in "100") does not appear in this sequence since the last two digits of S will never sum to 0, 00, or 100.

[Eric A.]

> ... Jim Nastos is right, sorry...
 > Shame on me...

[Andrew Weimholt]

If you modify the definition to...

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a(1) = 1
a(2) = 2
for n>2
a(n) = smallest unused natural number ending in sum mod 18 of last two digits of sequence, then
the new sequence IS a permutation of the naturals.
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The first difference in the two sequences is at $a(146)=20$ (vs 218) following $a(145)=99$.
 100 shows up at $a(1240)$ in the new sequence.

Thank you to all contributors.
 Best,
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