

Maple-assisted proof of empirical formula for A209042

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There are $4^3 = 64$ possible configurations for a row. Consider the 64×64 transition matrix T such that $T_{ij} = 1$ if the bottom row of a 2×3 sub-array could be in configuration i while the top row is in configuration j , and 0 otherwise. The following Maple code computes it. I'm encoding a row

$\begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix}$ as $b + 1$ where $b_1 b_2 b_3$ is the base-4 representation of b . The $+ 1$ is needed because matrix indices start at 1 rather than 0.

```
> q:= proc(a,b) local A,B;
    A:= convert(a-1+4^3,base,4) [1..3];
    B:= convert(b-1+4^3,base,4) [1..3];
    `if` (A[1] <> B[1] and `and` (seq((A[i-1]+B[i])/2<>A[i],i=2..3)
),1,0)
end proc:
T:= Matrix(4^3,4^3, q):
```

Thus $a(n) = u T^{n-1} v$ where u is the row vector of all 1's, and v the column vector corresponding to all allowed configurations for the top row (where we require $b_2 \neq b_1$ and $b_3 \neq b_2$).

```
> topro:= proc(a) local A; A:= convert(a-1+4^3,base,4) [1..3]; if A
[1] <> A[2] and A[2] <> A[3] then 1 else 0 fi end proc:
v:= Vector(4^3,topro):
u:= Vector[row](4^3,1):
```

To check, here are the first few entries of our sequence.

```
> V[1]:= v:
for n from 2 to 21 do V[n]:= T . V[n-1] od:
> seq(u . V[n], n = 1 .. 21);
36, 1300, 47700, 1752148, 64369764, 2364812680, 86878417604, 3191736905988,
117257942776700, 4307819083344444, 158260539240359944, 5814171346741659592,
213600867349885236600, 7847262801829583566600, 288292525423375500733688,
10591282886006506476586216, 389102259958583490236031192,
14294828146353910610243607696, 525163004079030666241727468760,
19293424029281428527910471040556, 708801282425516243300190864088356
```

Now here is the minimal polynomial P of T , as computed by Maple.

```
> P:= unapply(LinearAlgebra:-MinimalPolynomial(T, t), t);
P := t ↦ -6561 t + t40 + t39 - 899 t38 - 15561 t37 - 83213 t36 - 102927 t35 - 234538 t34
+ 426888 t33 + 6083455 t32 + 19226403 t31 + 30314632 t30 - 5616262 t29 - 91389155 t28
- 111865607 t27 + 12361268 t26 + 208623134 t25 + 135773863 t24 - 109627963 t23
- 40867993 t22 - 62635833 t21 - 102347812 t20 + 74400300 t19 - 159867 t18
+ 58817269 t17 + 84284989 t16 - 144160899 t15 - 6815858 t14 + 97275428 t13
- 72595286 t12 - 14152564 t11 + 68691483 t10 - 25250197 t9 - 23346963 t8
```

