W and M numbers

Definition (1):
A “W number” is an integer W with the property that every digit d_k of W is a sum of two or more distinct digits of W with indices different from k.

(Definition properly rephrased thanks to **Ed Jeffery**)

Example:
10070231 is a W number as:
- each digit “1” is the sum of another “1” and a “0”;
- each digit “0” is the sum of the two other “0”s;
- the digit “7” is the sum of the digits 1+2+3+1;
- the digit “2” is the sum of the digits 1+1;
- the digit “3” is the sum of the digits 2+1.

Sequence:
The W(n) sequence of “W numbers” starts like this:

W(n) = 10001, 10010, 10100, 11000, 20002, 20020, 20200, 22000, 30003, 30030, 30300, 33000, 40004, 40040, 40400, 44000, 50005, 50050, 50500, 55000, 60006, 60060, 60600, 66000, 70007, 70070, 70700, 77000, 80008, 80080, 80800, 88000, 90009, 90090, 90900, 99000, 100001, 100010, 100011, 100012, 100021, 100100, 100101, 100102, 100110, 100120, ...

Note:
I like the “W number” 10001124 as this is the smallest integer whose digits, mixed with the digits of any other integer, will produce a new “W number”. Take 69, for instance; concatenate 69 and 10001124 and you’ll get 6910001124; this is a “W number” (because “6” is the sum of 2+4 and “9” the sum of 1+1+1+2+4).

Exercise:
What is the smallest “W number” showing at least one “7”?

Prime sequence of “W numbers”:
The above 10070231 (“example”) is also a prime number. Is 102001 the smallest “W number” being a prime? How would the sequence of “Prime W numbers” look like?

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Definition (2):
An “M number” is an integer M with the property that every digit d_k of M is a product of two or more distinct digits of M with indices different from k.

Example:
16232131 is an "M number" as:
- each digit "1" is the product of the two other "1"s;
- the digit "6" is the product of "2" and "3";
- each digit "2" is the product of the other "2" and one "1";
- each digit "3" is the product of the other "3" and one "1";
- each digit "1" is the product of the two other "1"s.

Sequence:
The M(n) sequence of "M numbers" starts like this:

M(n) = 111, 1111, 10011, 10101, 10110, 11001, 11010, 11100, 11111, 11122, 11133, 11144, 11155, 11166, 11177, 11188, 11199, 11212, 11221, 11313, 11331, 11414, 11441, 11515, 11551, 11616, 11661, 11717, 11771, 11818, 11881, 11919, 11991, 12112, 12121, 12211, 13113, 13131, 13311, 14114, (...)

Note:
I like the "M number" 10011222335577 as this is the smallest integer whose digits, mixed with the digits of any other integer, will produce a new "M number". Take 69, for instance; concatenate 69 and 10011222335577 and you'll get 6910011222335577; this is an "M number" (because "6" is the product of the digits 2 and 3, and "9" the product 3*3).

Prime sequence of "M numbers":
The above 16232131 ("example") is also a prime number. Is 11177 the smallest "M number" being a prime? How would the sequence of "Prime M numbers" look like?

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January 5th update, 2012.

— "W numbers" are now in the OEIS (thanks to **Maximilian Hasler**), as A203591;
— The prime numbers of this sequence form A203592;
— **Charles Greathouse** comment on SeqFans was published yesterday:

> > So this sequence is 10-automatic: it can be recognized by a regular expression when written in decimal. I don't think it's feasible to write it out directly, though. But you can see that for every digit there is a collection of 1-5 multisets of digits, at least one of which needs to be contained in any number using the digit. For example, a number using the digit 2 must have either 00022 or 000112. For each of the 1023 combinations of digits, take the maximum of each involved digit to find the new multiset for that combination. From here it's obvious that a regular expression exists, as well as that it must be large since each order of digits in each multiset must be encoded.

It’s a pity, though, since an explicit finite automaton (from the regex) would give a closed-form expression for the number of d-digit members. It’s about 0.9*10^d, of course.

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Thank you to all contributors,
Best,
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Brussels, January 3rd 2012.