

I independently came up with an alternative way to obtain the sequence A182541 and it allows me to suggest a possible interpretation for it. I can't prove in a formal way that the sequence suggested by me and A182541(n) are, in fact, the same, however, my formula correctly calculates all the listed terms for A182541(n) to consider it just a mere coincidence.

So let's consider the number of permutations of 1...n that have 2 following 1 for $n \geq 1$

(A001710) Jon Perry 2008

The sequence of matrices can be constructed so that all the rows consist of permutations of 11...n

| | | | |
|-----|-------|------|-------|
| 11, | 112 , | 1123 | 11234 |
| | 121 | 1132 | 11243 |
| | 211 | 1231 | |
| | | 1213 | |
| | | | |

Then for each matrix we are going to do the following. For each element with the value of 'k' delete exactly k elements of this type from the matrix, repeat again if necessary until no elements with the value of k are left and the last element is reached. The whole operation should be repeated for all the values of k from 1 to n until no more elements can be deleted:

11 --> 1 1 element is left and there are no more ones to delete
=> n(1) = 1

| | | | |
|---------|-----|-----|--|
| 112 | 102 | 102 | |
| 121 --> | 120 | -> | 100 4 non-zero elements are left => n(2)=4 |
| 211 | 210 | 010 | |

A182541(n) basically counts all the non-zero elements left in the matrix after the procedure of "deletion of elements" was completed as it was defined above.

The deletion can be done in any order:

| | | | |
|--------------|------|----------|---------------|
| 1123 | 1123 | 1123 | 1023 |
| 1132 | 1102 | 1100 | 1000 |
| 1231 | 1201 | 1001 | 1000 |
| 1213 | 1210 | 1210 | 1200 |
| 1321 | 1321 | 1301 | 1300 |
| 1312 del3 -> | 1012 | del 2--> | 1010 del 1 -> |
| 2113 | 2110 | 2110 | 2100 |
| 2131 | 2101 | 0101 | 0100 |
| 3112 | 3112 | 3110 | 3100 |
| 3121 | 0121 | 0121 | 0120 |
| 3211 | 0211 | 0011 | 0010 |
| 2311 | 2011 | 0011 | 0010 |

19 elements are left. n(3) = 19

Clearly there is no need to draw all these matrices to get the A182541(n) as the number of rows of any matrix can be obtained directly from A001710 as the number of permutations.

Finally, A182541(n) can be calculated by the means of formula suggested by me. These matrices have a special property, so that

[The Number of rows] = 0 (mod[the number of k-elements left]). for every k = 1 to n.

Initially each matrix has the same number of elements bigger than 1 and it equals the number of its rows. It also has the number of elements with a value of 1 equal to the (number of rows)*2.

After the procedure of deletion of the elements is finished we got the number of 'ones' which is equal to the number of rows in the matrix. the number of twos which is equal to the (number of rows)/3 the number of threes which is equal to the (number of rows)/4 and so on.

Basically, by definition,

$$a(1) = 1$$

$$a(2) = \text{The number of non-zero elements left} = \text{Sum of all the numbers of all the non-zero elements left.} = \text{the number of all the "1-elements" left} + \text{the number of all the "2-elements" left} = \text{Number of rows} + \text{Number of rows} / 3 = 3 + 3/3 = 3 + 1 = 4$$

$$a(3) = \text{number of all the "1-elements" left} + \text{number of all the "2-elements" left} + \text{number of all the "3-elements" left} = \text{Number of rows} + \text{Number of rows} / 3 + \text{Number of rows} / 4 = 12 + 12/3 + 12/4 = 19. \text{ and so on.}$$

$$a(1) = 1$$

$$\text{for } n \geq 2 \quad a(n) = A001710(n+1) * [1 + \text{Sum}_{k=2..n} 1/(k+1)]$$

and we get the sequence:

1, 4, 19, 107, 702, 5274, 44712, 422568 , ...

The only noticeable difference between the two that can be seen is that A182541(n) defined by Sergey Kitaev, Jeffrey Remmel starts from a(3) = 1

and the 'Matrix-related' or 'sum of the quotients of A001710' sequence defined by me starts from a(1) = 1 what I believe may be even a bit more relevant, but anyway...

for Coefficients in g.f. for certain marked mesh patterns. the formula will look like:

$a(3)=1$ For $n \geq 4$ $a(n)=(A001710(n-1))*[1+ \text{Sum}_{k=2..n-2} 1/(k+1)]$

but it is the exact same.

Lastly, I want to mention one separate property.

Applying this rather odd or clumsy operation of 'deletion of the elements' to the similar type of matrices based on the sequence

112, 1123, 11234, 11235

We get the Central Polygonal numbers. (OEIS A000124)

| | | |
|------|------|----------|
| 1113 | | 1013 |
| 1131 | ---> | ... 0100 |
| 1311 | | 1001 |
| 3111 | | 0010 |

7 non-zero elements are left and it is the Central Polygonal number for $n=3$

11114

.....
.....
.....

41111 gives us 11 or the Central Polygonal number for $n=4$ and so on

Of course, it is rather trivial to be considered any seriously and I added it just mostly an illustration purpose only to show that this operation may indeed have some sense/value.
