Rademacher's Infinite Partial Fraction Conjecture is (almost certainly) False

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The first-named author's "academic grandfather", Hans Rademacher, was a great number theorist, but even great mathematicians sometimes make false conjectures. In this article we prove (empirically) that a conjecture made by Rademacher in his posthumously published classic "Topics in Number Theory" is (very!) false as stated, but if you replace "infinity" by some good-old finite numbers it may be resurrected.

Maple Package

Important: This article is accompanied by Maple package

- **HANS**
  [Added March 13, 2012: This new version of HANS contains a new procedure, E01s. For the record, here is the old version of HANS]

Sample Input and Output for HANS

- **If you want to see the first 700 terms of the sequence $C_{011}(N)$ as exact rational numbers, followed by their floating-point renditions, that overwhelmingly shatter Rademacher's conjecture by showing that that sequence does *not* converge to anything (in particular not to -0.29292754...) but instead eventually oscillates widely getting ever-so-clse to plus infinity and ever-so-close to negative infinity (with a period that seems to be 32), the input gives the output.**

- **If you want to see the first 500 terms of the sequences $C_{01j}(N)$ for j from 1 to 10, both in exact rational arithmetic, and in**

**floating-point,
the input gives the output.**

- **If you want to see the first 700 terms of the sequence $C_{121}(N)$
  the input gives the output.**

- **If you want to see the "closest encounters" of the sequences $C_{hkl}(N)$ to Radmacher's alleged (but wrong!) "limit" (that he called, with wishful thinking, $C_{hkl}(\infty)$) for $0 \le h < k \le 3$, (gcd(h,k)=1), $l \le 5$, and $N \le 250$,
  the input gives the output.**

- **If you want to conduct your own computer experiments with our data, we have put all the 10 sequences $C_{01j}(N)$ for $1 \le j \le 10$, for $1 \le N \le 800$, into one file, called
  HANSC01,
  in Maple readable format. We named that**

**sequence C01r. For example, $C_{017}(597)$ could be gotten (once you uploaded that file), by typing
C01r[7][597];**

- **An even larger list then above, put all the 10 sequences $C_{01j}(N)$ for $1 \le j \le 10$, for $1 \le N \le 850$, into one file, called
[HANSC01a](),
in Maple readable format. We also named that sequence C01r. For example, $C_{017}(597)$ could be gotten (once you uploaded that file), by typing
C01r[7][597];**

- **If you want even more data, but in floating-point, we put all the 40 sequences $C_{01j}(N)$ for $1 \le j \le 40$, for $1 \le N \le 1000$, into one file, called
[HANSC01f](),
in Maple readable format. We named that sequence C01f. For example, the floating-**

point approximation of $C_{017}(999)$ could be gotten gotten (once you uploaded that file), by typing C01f[7][999];

- **If you want to see the 21 sequences $C_{01(-j)}(N)$ for j from 0 to 20 and $1 \le N \le 500$ the [input](#) yields the [output](#) , in Maple readable format. We named that sequence C01Minus. To get $C_{01(-j)}(N)$, simply type, C01Minus[j+1][N]; For example, $C_{01(-7)}(456)$ could be gotten (once you uploaded that file), by typing C01Minus[8][456];**

- **If you want to see conjectured (appx.) asymptotic expressions for $C_{01l}(N)$ for l between 1 and 15, the [input](#) gives the [output](#).**

- **If you want to see the values, in floating-point, of $C_{hkl}(N)$ for $0 < h < k < 10$, $k \ge 3$,**

with gcd(h,k)=1 and for l between 1 and 10, and N between 1 and 400
the input gives the output.

- **[Added March 13, 2012]**

  If you want to see the first 1500 terms of the sequence C_{011}(N)*(2*N)!,
  the input gives the output.

  [Added April 17, 2012]

  If you want to see the first 2000 terms of the sequence C_{011}(N)*(2*N)!,
  the input gives the output.

- **[Added March 13, 2012]**

  If you want to see conjectured (appx.) asymptotic expressions for C_{011}(N) (using 1500 terms rather than 900 as in oHANS10)
  the input gives the output.

[Added April 17, 2012]

If you want to see conjectured (appx.) asymptotic expressions for $C_{011}(N)$ (using 2000 terms rather than 1500 as in oHANS13)
the input gives the output.
[Note the "shifting of the perihelion!", now the maxima are at 1(mod 32) and the minima at 17 (mod 32)]

Doron Zeilberger's List of Papers

Doron Zeilberger's Home Page