Maple-assisted derivation of recurrence for A181252

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There are $2^9 = 512$ possible rows. We enumerate them so that row i consists of the binary digits of $t - 1$ (with leading 0's included as needed).

Consider the 512×512 transition matrix T with entries $T_{ij} = 1$ if the rows of a 2×9 sub-array could be in configurations i and configuration j, and 0 otherwise. The following Maple code computes it.

```
> for i from 1 to 512 do Configs[i]:= convert(2^9+i-1,base,2)[1..9]
  od:
```

```
> Compatible:= proc(i,j)
   if `and`(seq(evalb(Configs[i][k] + Configs[i][k+1] + Configs[j]
  [k]+Configs[j][k+1] < 4), k=1..8)) then 1 else 0 fi
  end proc:
  T:= Matrix(512,512,Compatible):
```

Thus for $n \geq 1$ $\quad a(n) = v^T T^{n-1} v$ where v is a column vector with all entries 1.

```
> v:= Vector(512,1):
```

To check, here are the first few entries of our sequence. For future use, we pre-compute more $T^n u$ than we need.

```
> TV[0]:= v:
  for nn from 1 to 150 do TV[nn]:= T . TV[nn-1] od:
  seq(v^%T . TV[n],n=0..23);
```

512, 169209, 61986457, 22161786304, 7969215344753, 2861765993703849, 1027999596778673856, 369248337357375835969, 132633131268024896655873, 47641303155727829675539968, 17112587585474467714330327353, 6146779451170285748070586194193, 2207901031429162144096022660047872, 793070084470057302384087155377692617, 284867913890868281060496009223548603177, 102323527128117748231683605598645592696000, 36754241861857942427618829272595113039990337, 13201991103089617015868946418191543366328499257, 4742107584286728066238654598555598612166175562176, 1703347939359914455191399817639020500175376703279249, 611836435794344642814998651901513812148245981026094097, 219769440826134343472025034384093298118025874905674197504, 78940390430209402988161326956102126571047543727131812487561, 28355103502328276775783627431590611233398261842020833876409633 **(1)**

The recurrence shows up as a linear dependence among $T^n v$. We gather these as columns of a matrix L, and stop when it has less than full column rank.

```
> L:= v:
  for nn from 1 do
    L:= <L|TV[nn]>;
```

```
    if LinearAlgebra:-Rank(L) < nn+1 then printf("Success at n=
  %d\n",nn); break fi;
  od:
Success at n=48
```

The recurrence can then be found from the null space of the matrix L.

```
> P:= LinearAlgebra:-NullSpace(L)[1]:
```

This is the recurrence:

```
> recurrence:= sort(a(n)=solve(add(P[i]*a(n+i-49),i=1..49),a(n)),
  [seq(a(n-i),i=0..49)]);
```

$$
\begin{aligned}
recurrence := a(n) = {} & 238\, a(n-1) + 47314\, a(n-2) - 753400\, a(n-3) - 233197418\, a(n-4) + 4640749138\, a(n-5) + 445768132358\, a(n-6) - 15947750747086\, a(n-7) \\
& - 166397080885738\, a(n-8) + 16044964361788296\, a(n-9) \\
& - 232268164267011902\, a(n-10) - 1259342300696935690\, a(n-11) \\
& + 56703476089033781841\, a(n-12) - 175163843594441853916\, a(n-13) \\
& - 6158486425988745282892\, a(n-14) + 37828515864772141827536\, a(n-15) \\
& + 426730518403164436524924\, a(n-16) - 3187764949147877154239276\, a(n-17) \\
& - 22263015848318609094872172\, a(n-18) + 156817786076227057766175044\, a(n-19) + 928402921982255234672429452\, a(n-20) \\
& - 4732733396403067821956407920\, a(n-21) \\
& - 29411828959052712955794937564\, a(n-22) \\
& + 81383986022203958050802246732\, a(n-23) \\
& + 635269522936784233210087183281\, a(n-24) \\
& - 599879731383151109663500771338\, a(n-25) \\
& - 8443802805789621505270613872518\, a(n-26) \\
& - 1254953806327313941362136762072\, a(n-27) \\
& + 6694250178093943741260209312404 6\, a(n-28) \\
& + 43748051598047626043299145788986\, a(n-29) \\
& - 345972425459159767821217962670458\, a(n-30) \\
& - 279451872740517323160418400653014\, a(n-31) \\
& + 1261296152458153574908530105330750\, a(n-32) \\
& + 882589876903507307806480248540648\, a(n-33) \\
& - 3352816000524260245832271794017350\, a(n-34) \\
& - 1343294873274359177108950077340002\, a(n-35) \\
& + 6296023406954446417610451728434143\, a(n-36) \\
& + 873552059605699295620890073181 04\, a(n-37) \\
& - 7455787668810085037129172828741600\, a(n-38) \\
& + 2791482728098756463529553428371712\, a(n-39) \\
& + 4228172599624392603097534465319040\, a(n-40) \\
& - 3277612624942382391365575708802880\, a(n-41) \\
& - 181873431853321055758500253305600\, a(n-42) \\
& + 741628402902875277367457752166400\, a(n-43) \\
& - 105194093277546086438275006464000\, a(n-44) \\
& - 59902648177113781255816151040000\, a(n-45)
\end{aligned} \tag{2}
$$

$$+ 10650559846199432301898629120000\, a(n-46)$$
$$+ 1806046600239285875968573440000\, a(n-47)$$
$$- 174135216999916792656691200000\, a(n-48)$$