Proof for: A238443 = A174973 = ¬A238542

Hartmut F. W. Höft
2014-12-01

Notations

Let n = $2^m \times q$ where m≥0, q is odd and has the prime factorization q = $\prod_{i=1}^{k} p_i^{e_i}$ for primes $2 < p_1 < \ldots < p_k$ and exponents $e_i > 0$, for all 1 ≤ i ≤ k. Let $q_j = \prod_{i=1}^{j} p_i^{e_i}$, for 0 ≤ j ≤ k.

In this notation, the condition by David S. Metzler for numbers n to be in A174973 (see the Comment section) can be written as: $p_j \leq 2^{m+1} \times q_{j-1}$, for 1 ≤ j ≤ k. (*)

Let r_n = A003056(n) be the length of the n-th row of the irregular triangle A237048 of odd divisors of n. If s < r_n is an odd divisor of n and n = 2^m×s×t, then A237048(n, s) = A237048(n, 2^{m+1}× s) = 1.

Claim #1 n ∉ A174973 ⇒ n ∈ A238542

Using property (*), there is a smallest j such that $p_j > 2^{m+1} \times q_{j-1}$. Then for all odd divisors $s < p_j$ of n, also $2^{m+1} \times s < p_j$. The observation above implies that the numbers of 1's at odd indices s and even indices of form $2^{m+1} \times s$ are matched, and that therefore the width of the symmetric representation of σ(n) at entry A249223(n, 2^{m+1}× s), for some s, is zero, ensuring at least two parts, i.e. n ∈ A238542.

Claim #2 n ∈ A238542 ⇒ n ∉ A174973

Since the symmetric representation of σ(n) has at least two parts, there is an index u = $2^{m+1} \times s$ such that A249223(n, u) = $\sum_{\alpha=1}^{u} (-1)^{\alpha+1} \times$ A237048(n, α) = 0. Hence there is an odd t with n = $2^m \times s \times t$. Since $2^{m+1} \times s \leq r_n$ we have r_n < t, i.e. an odd divisors larger than s. Suppose that x is an odd divisor larger than s. Then $2^{m+1} \times s <$ x since an entry of 1 in A238048 for each odd divisor z < $2^{m+1} \times s$ must be matched by a 1 on an index of form $2^{m+1} \times z$. If odd number x is the next divisor larger than divisor $2^m \times s$ then $2^m \times s < 2^{m+1} \times s <$ x , and n ∉ A174973. Let the next divisor z larger than $2^m \times s$ be even, i.e., z = $2^j \times y$ with 0 < j ≤ m and y odd. If z ≤ $2^{m+1} \times s$ we get s ≤ $2^{m-j} \times s < y <$ z = $2^j \times y < 2^{m+1} \times s$, but that is impossible since y is odd. Therefore $2^{m+1} \times s <$ z , and n ∉ A17497.