We prove that binomial(n + 10*k!, k) = binomial(n,k) + 10q where q is an integer.

We consider the polynomial $P(x) = (n+x)(n-1+x)(n-2+x)...(n-k+1+ x)$.

The constant term is :

$n(n-1)(n-2)...(n-k+1)$.

There exists a polynomial $Q(x)$ such that :

$P(x) = n(n-1)(n-2)...(n-k+1)+ xQ(x)$.

We have :

$$\text{binomial}(n+10k!\ ,\ k) = \frac{(n + 10k!)(n - 1 + 10k!)(n - 2 + 10k!)...(n - k + 1 +10k!)}{k!}$$

The numerator of this expression is $P(10k!)$. Then, we obtain :

$$\text{binomial}(n+10k!\ ,\ k) = \frac{n(n - 1)(n - 2) ...(n - k + 1) + 10k!\ Q(10k!)}{k!}$$

$$= \frac{n(n - 1)(n - 2) ...(n - k + 1)}{k} + 10Q(10k!)$$

Let $q = Q(10k!)\ k$, then :

binomial(n+10k! , k) = binomial(n+10k! , k) + 10q

from this result, we have proved that $a(n) + 10k\ ! = a(n)$.