

Sequence **A169720**, and two others by Alice V. Kleeva

This page illustrates a few mysterious and confusing OEIS sequences submitted in 2010 by Alice V. Kleeva.

Contents

[A169720](#)

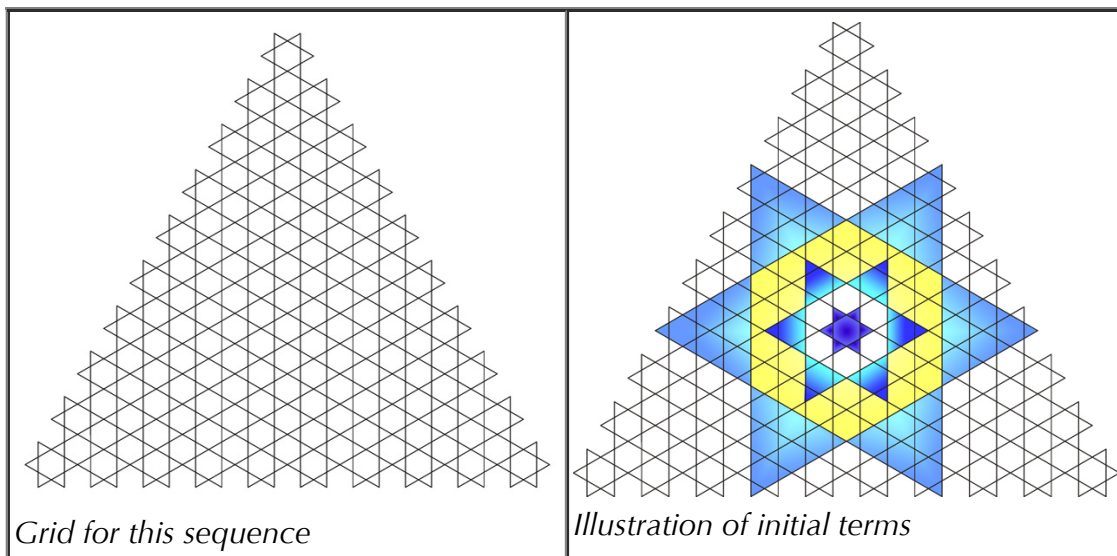
[A169727](#)

[A169726](#)

A169720

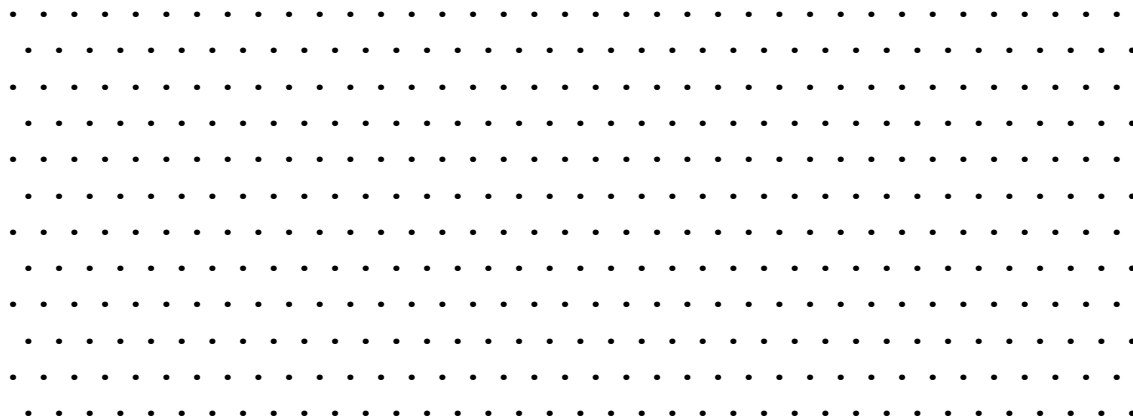
This sequence, Sloane's [A169720](#), includes some of the [triangular numbers](#) identified with a certain geometrical grid construction by Alice Kleeva of the Hermitage Museum in St. Petersburg. You may read her description (as paraphrased by Neil Sloane) [here on seqfan](#).

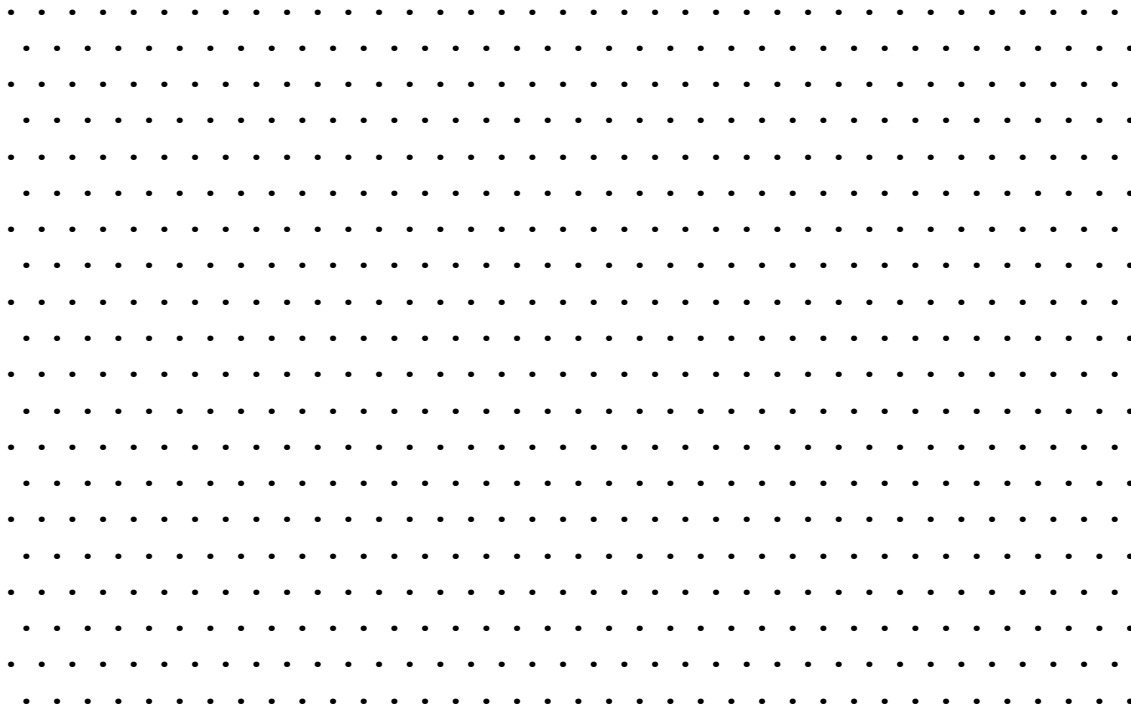
The sequence **A169720** begins: 1, 10, 55, 253, 1081, ... By means of illustration, the author provided the following figures:



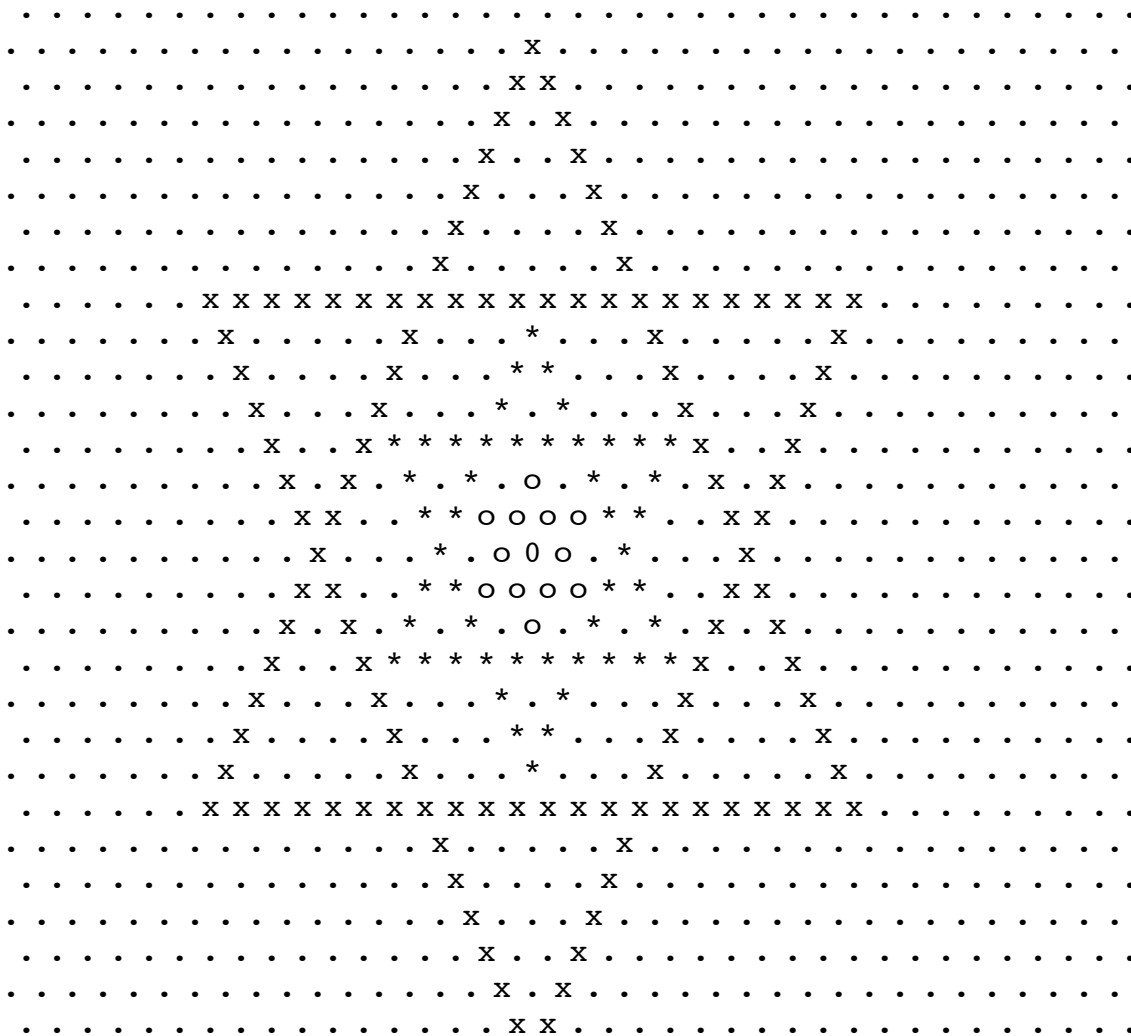
The figures provide too much, and at the same time too little, explanation. The sequence is actually constructed as follows:

1. Start with the central points of a hexagonal grid, or the vertices of a triangular grid:





2. Draw concentric 6-pointed star (also known as "Star of David") figures in the grid. Make each star just large enough so that its central hexagon surrounds, but does not intersect, the 6-pointed star within its center:



```

. . . . . X . . . . .
. . . . .

```

(The first 6-pointed star is represented by a single point in the center; the next one consists of the vertices labeled "o"; the next consists of "*" , the next consists of "x", and there would be more larger ones around that.)

3. Replace each star with a filled triangle and count the number of points in the triangle:

```

. . . . . 0 . . . . .
. . . . .
A[1] = 1

```

```

. . . . . O . . . . .
. . . . . O O . . . . .
. . . . . O O O . . . . .
. . . . . O O O O . . . . .
. . . . .
A[2] = 10

```

```

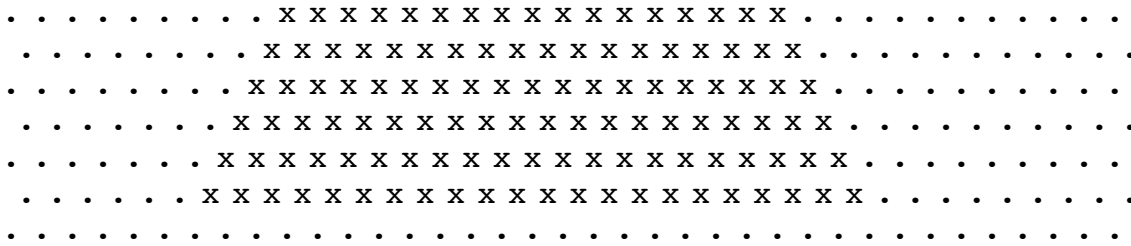
. . . . . * . . . . .
. . . . . * * . . . . .
. . . . . * * * . . . . .
. . . . . * * * * . . . . .
. . . . . * * * * * . . . . .
. . . . . * * * * * * . . . . .
. . . . . * * * * * * * . . . . .
. . . . . * * * * * * * * . . . . .
. . . . . * * * * * * * * * . . . . .
. . . . .
A[3] = 55

```

```

. . . . . X . . . . .
. . . . . X X . . . . .
. . . . . X X X . . . . .
. . . . . X X X X . . . . .
. . . . . X X X X X . . . . .
. . . . . X X X X X X . . . . .
. . . . . X X X X X X X . . . . .
. . . . . X X X X X X X X . . . . .
. . . . . X X X X X X X X X . . . . .
. . . . . X X X X X X X X X X . . . . .
. . . . . X X X X X X X X X X X . . . . .
. . . . . X X X X X X X X X X X X . . . . .
. . . . . X X X X X X X X X X X X X . . . . .
. . . . . X X X X X X X X X X X X X X . . . . .

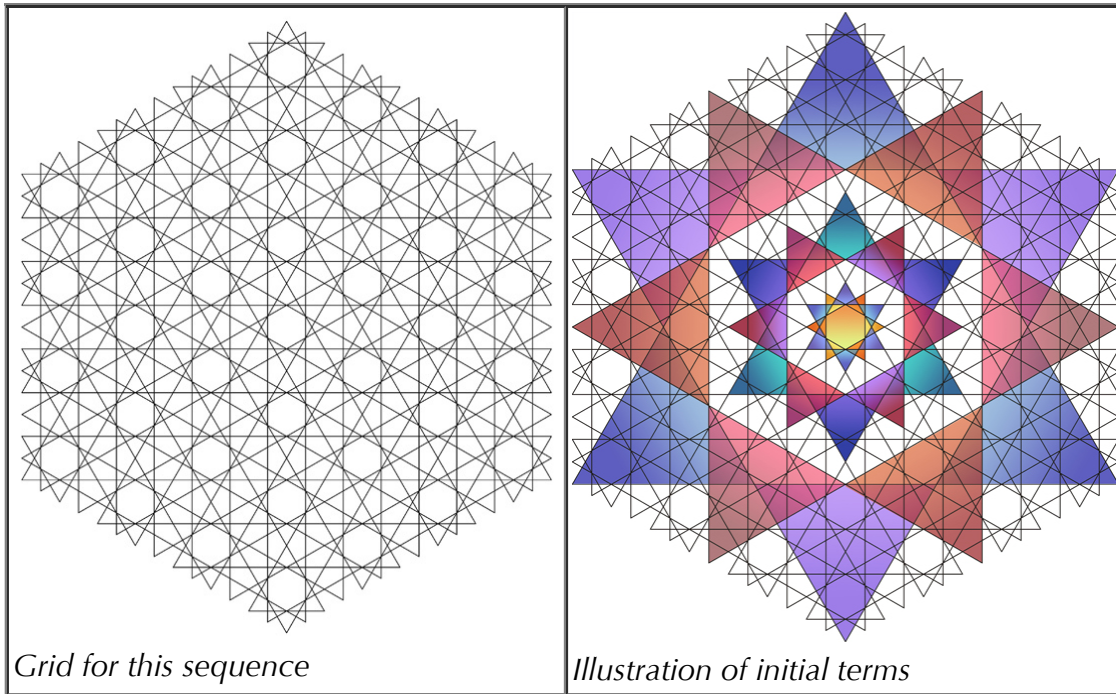
```



$$A[4] = 253$$

A169727

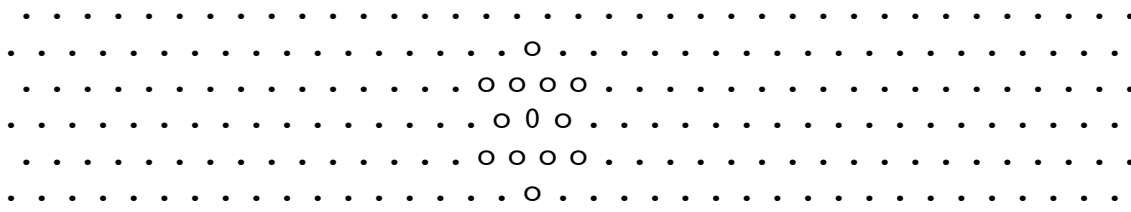
Sequence [A169727](#) is illustrated by two similarly uninformative figures:



The sequence starts: 1, 19, 127, 631, 2791, ... and is a subsequence of [A003215](#).

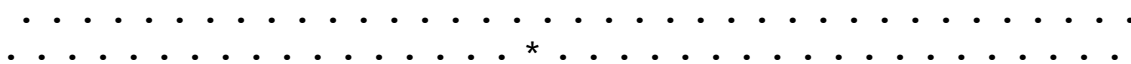
Contrary to the seeming complexity of the figures, the sequence is constructed almost the same way as [A169720](#), with a series of concentric 6-pointed stars on a grid. Only the third step is different:

Step 3 for [A169727](#). Count the number of points inside each star's central hexagon:



(1st star, interior has just one 'O')

$$A[1] = 1$$



```

. . . . . * * . . . . .
. . . . . * * . . . . .
. . . . . * * * * * * * * * * . . . . .
. . . . . * * O O O * * . . . . .
. . . . . * * O O O O * * . . . . .
. . . . . * O O O O O * . . . . .
. . . . . * * O O O O * * . . . . .
. . . . . * * O O O * * . . . . .
. . . . . * * * * * * * * * * . . . . .
. . . . . * * . . . . .
. . . . . * * . . . . .
. . . . . * . . . . .

```

(2rd star, interior filled with 'o')
 There are 19 o's. A[2] = 19

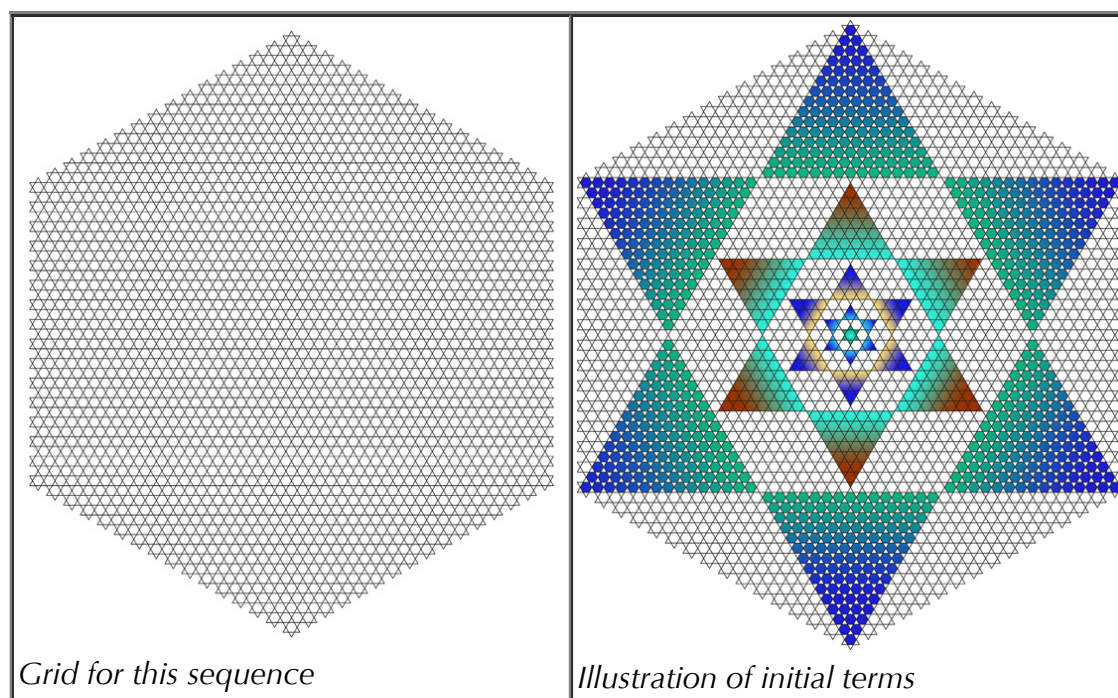
```

. . . . . X . . . . .
. . . . . X X . . . . .
. . . . . X . X . . . . .
. . . . . X . X . . . . .
. . . . . X . . X . . . . .
. . . . . X . . . X . . . . .
. . . . . X . . . . X . . . . .
. . . . . X X X X X X X X X X X X X X X X X X X X X X X X . . . . .
. . . . . X . . . X * * * * * * * X . . . . X . . . . .
. . . . . X . . . X * * * * * * * * X . . . . X . . . . .
. . . . . X . . . X * * * * * * * * * X . . . . X . . . . .
. . . . . X . X * * * * * * * * * * X . X . . . . .
. . . . . X X * * * * * * * * * * * X X . . . . .
. . . . . X * * * * * * * * * * * * X . . . . .
. . . . . X X * * * * * * * * * * * X X . . . . .
. . . . . X . X * * * * * * * * * * X . X . . . . .
. . . . . X . . X * * * * * * * * * * X . . . . X . . . . .
. . . . . X . . . X * * * * * * * * X . . . . X . . . . .
. . . . . X . . . X * * * * * * * * X . . . . X . . . . .
. . . . . X . . . . X * * * * * * * X . . . . X . . . . .
. . . . . X X X X X X X X X X X X X X X X X X X X X X X X X X . . . . .
. . . . . X . . . . X . . . . .
. . . . . X . . . . X . . . . .
. . . . . X . . . . X . . . . .
. . . . . X . . . . X . X . . . . .
. . . . . X X . . . . .
. . . . . X . . . . .

```

(3rd star, interior filled with '*')
 There are 127 *'s. A[3] = 127

Sequence [A169726](#), once again, has two confusing figures:




The sequence starts: 1, 7, 37, 169, 721, 2977, ... and like **A169727** it is a subset of [A003215](#).

I won't bother to illustrate the construction, but it is very similar to that for **A169727**, again we are counting the points inside the central hexagon of each in a series of concentric 6-pointed stars. These central hexagons are white in the second figure above.

As it turns out, the terms in this sequence are just the next term (from **A003215**) after those **A003215** terms that appear in **A169727**. In other words:

1. Start with **A169727**, which is: 1, 19, 127, 631, 2791, ...
2. Replace each term with the next term from [A003215](#). 1 becomes 7; 19 becomes 37; 127 becomes 169; 631 becomes 721; and so on. We get: 7, 37, 169, 721, 2977, ...
3. Add an initial 1. Now we have **A169726**: 1, 7, 37, 169, 721, 2977, ...

Footnotes

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