

***k*-comma numbers**

(or *commatiles* years)

(version française abrégée tout en bas)

Let $a(1) = 0$

$a(2) = 1$

and $a(n) = a(n-1) +$ [the two-digit integer split by the comma which separates $a(n-1)$ and $a(n-2)$]

The first 100 terms of the sequence are (computed by **Farideh Firoozbakht**):

S = 0, 1, 2, 14, 35, 78, 135, 216, 268, 330, 413, 417, 451, 525, 540, 595, 600, 656, 662, 728, 755, 842, 900, 929, 938, 1037, 1118, 1189, 1270, 1361, 1362, 1373, 1394, 1425, 1466, 1517, 1578, 1649, 1730, 1821, 1822, 1833, 1854, 1885, 1926, 1977, 2038, 2110, 2192, 2194, 2216, 2258, 2320, 2402, 2404, 2426, 2468, 2530, 2612, 2614, 2636, 2678, 2740, 2822, 2824, 2846, 2888, 2950, 3032, 3035, 3058, 3111, 3194, 3207, 3250, 3323, 3326, 3359, 3422, 3515, 3538, 3591, 3674, 3687, 3730, 3803, 3806, 3839, 3902, 3995, 4018, 4072, 4156, 4180, 4244, 4248, 4292, 4376, 4400, 4464, ...

14 is in **S** because we add to 2 the integer 12 seen around the comma of [1,2]

35 is in **S** because we add to 14 the integer 21 seen around the comma of [2,14]

417 is in **S** because we add to 413 the integer 4 seen around the comma of [330,413]

Note:

I am looking for "comma numbers", which are numbers like [abc] where the split [a,bc] or the split [ab,c] would produce later in the sequence the said "comma number" again: < ..., abc, ... >

Example: 416 is not a "comma number" because we have no hit for the two different starts [4,16] or [41,6]:

$S_1 = 4, 16, 57, 122, 193, 214, 246, 288, 350, 433, \dots$ <-- no hit

$S_2 = 41, 6, 22, 84, 112, 153, 174, 205, 247, 299, 371, 464, \dots$ <-- no hit

I guess 10 is the first comma number:

10 --> 1,0,10,11,12,23, etc. --> yes '10' is in the development of [1,0], thus '10' is a comma number

(we see immediately that '20', '30', '40', and a few more are comma numbers too)

What about 11?

11 --> 1,1,12,23, etc. <-- no hit: '11' is not a comma number

How does the "comma numbers sequence" look like?

Best,
É.

(This was inspired by [A121805](#): « The "commas" sequence: $a(1) = 1$; for $n > 1$, let x be the least significant digit of $a(n-1)$; then $a(n) = a(n-1) + x*10 + y$ where y is the most significant digit of $a(n)$ and is the smallest such y , if such a y exists. If no such y exists, stop. »

Maximilian Hasler was quick to compute the sequence:

$S_{\text{comma numbers}} = 10, 20, 30, 40, 50, 60, 70, 80, 90, 109, 123, 148, 160, 163, 185, 198, 199, 241, 340, 362, 398, 422, 423, 444, 522, 540, 541, 550, 564, 597, 621, 622, 667, 683, 693, 724, 769, 770, 780, 806, 811, 835, 842, 888, 972, 1023, 1044, 1053, 1054, 1116, 1146, 1177, 1187, 1238, 1248, 1299, 1310, 1329, 1362, 1391, 1394, 1410, 1424, 1475, 1482, 1483, 1491, 1493, 1501, 1504, 1513, 1521, 1535, 1576, 1578, 1585, 1606, 1657, 1732, 1779, 1789, 1799, 1833, 1840, 1852, 1860, 1861, 1873, 1880, 1925, 1926, 1952, 1963, 1984, 1986, 1994, 1996, 2005, 2036, 2067, 2109, 2150, 2171, 2191, 2232, 2234, 2243, 2256, 2258, 2263, 2273, 2287, 2307, 2317, 2359, 2404, 2414, 2422, 2426, 2431, 2441, 2442, 2444, 2446, 2451, 2466, 2523, 2533, 2567, 2570, 2587, 2612, 2619, 2621, 2636, 2639, 2646, 2654, 2656, 2676, 2711, 2718, 2750, 2780, 2783, 2793, 2795, 2824, 2827, 2834, 2844, 2847, 2899, 2950, 2951, 2990, 3035, 3043, 3056, 3066, 3075, 3076, 3098, 3151, 3152, 3162, 3225, 3234, 3238, 3247, 3268, 3278, 3290, 3326, 3346, 3399, 3414, 3437, 3440, 3450, 3460, 3470, 3513, 3515, 3516, 3555, 3579, 3589, 3602, 3631, 3652, 3745, 3748, 3768, 3801, 3811, 3821, 3879, 3894, 3942, 4051, 4072, 4113, 4139, 4149, 4156, 4159, 4193, 4203, 4213, 4311, 4315, 4321, 4331, 4383, 4395, 4429, 4463, 4512, 4555, 4557, 4567, 4577, 4585, 4592, 4596, 4599, 4611, 4653, 4685, 4689, 4733, 4743, 4761, 4781, 4825, 4827, 4847, 4871, 4923, 4939, 4945, 4949, 4952, 4969, ...$

Is your year of birth in the sequence? In a private mail **Maximilian** coined those integers as "commatile years". We were soon wondering if some of those integers could be doubly "comma" (or 2-comma, or "bicommatile"). An integer $\langle abcd \rangle$, for instance, could come back to life via two different splits, like $|a, bc|$ or $|ab, c|$. **Maximilian** had the answer:

Bicommatile years 0 \rightarrow 10000 (or 2-comma numbers):

1023, 1044, 1521, 1657, 1789, 1984, 2191, 2263, 2451, 2466, 2523, 2676, 2783, 2824, 3066, 3268, 3589, 3602, 3631, 4051, 4113, 4149, 4159, 4213, 4315, 4611, 4685, 4781, 4969, 5133, 5526, 6053, 6165, 6246, 6445, 6650, 6712, 6893, 7350, 7668, 8011, 8144, 8311, 8349, 8475, 8538, 8657, 8740, 8755, 8760, 8929, 8947, 9183, 9195, 9319, 9427, 9664, 9782, 9811, 9863, 9980, ...

Tricommatile years 0 → 30000 (or 3-comma numbers):

2676, 6246, 9427, 10587, 11558, 11756, 11811, 12427, 12788,
13090, 13110, 14328, 15031, 15187, 15493, 15637, 16867, 18322,
18768, 19918, 20699, 21138, 21422, 21698, 22824, 23108, 23242,
23868, 24456, 24854, 25342, 25478, 26583, 26948, 27883, 28418,
29538, 29986, ...

Development of the "proof" for the first 5-digit term above,
10587:

[**1058,7**], (+87=) 94, (+79=) 173, 214, 246, 288, 350, 433, 437,
471, 545, 560, 615, 621, 677, 693, 769, 806, 904, 973, 1022,
1053, 1074, 1105, 1146, 1197, 1258, 1329, 1410, 1501, 1502,
1513, 1534, 1565, 1606, 1657, 1718, 1789, 1870, 1961, 1962,
1973, 1994, 2025, 2067, 2119, 2191, 2283, 2295, 2327, 2379,
2451, 2543, 2555, 2587, 2639, 2711, 2803, 2815, 2847, 2899,
2971, 3063, 3076, 3109, 3172, 3265, 3288, 3341, 3424, 3437,
3480, 3553, 3556, 3589, 3652, 3745, 3768, 3821, 3904, 3917,
3960, 4033, 4037, 4071, 4145, 4159, 4213, 4307, 4341, 4415,
4429, 4483, 4577, 4611, 4685, 4699, 4753, 4847, 4881, 4955,
4969, 5023, 5118, 5153, 5238, 5273, 5358, 5393, 5478, 5513,
5598, 5633, 5718, 5753, 5838, 5873, 5958, 5993, 6078, 6114,
6200, 6246, 6252, 6318, 6344, 6430, 6476, 6482, 6548, 6574,
6660, 6706, 6712, 6778, 6804, 6890, 6936, 6942, 7008, 7035,
7122, 7179, 7206, 7303, 7370, 7407, 7414, 7491, 7538, 7555,
7642, 7699, 7726, 7823, 7890, 7927, 7934, 8011, 8059, 8077,
8175, 8253, 8311, 8349, 8367, 8465, 8543, 8601, 8639, 8657,
8755, 8833, 8891, 8929, 8947, 9045, 9124, 9183, 9232, 9271,
9300, 9319, 9328, 9427, 9516, 9595, 9664, 9723, 9772, 9811,
9840, 9859, 9868, 9967, 10056, 10127, 10188, 10259, 10340,
10431, 10432, 10443, 10464, 10495, 10536, (+51=) 10587.

[**105,87**], (+58=) 145, (+71=) 216, 268, 330, 413, 417, 451,
525, 540, 595, 600, 656, 662, 728, 755, 842, 900, 929, 938,
1037, 1118, 1189, 1270, 1361, 1362, 1373, 1394, 1425, 1466,
1517, 1578, 1649, 1730, 1821, 1822, 1833, 1854, 1885, 1926,
1977, 2038, 2110, 2192, 2194, 2216, 2258, 2320, 2402, 2404,
2426, 2468, 2530, 2612, 2614, 2636, 2678, 2740, 2822, 2824,
2846, 2888, 2950, 3032, 3035, 3058, 3111, 3194, 3207, 3250,
3323, 3326, 3359, 3422, 3515, 3538, 3591, 3674, 3687, 3730,
3803, 3806, 3839, 3902, 3995, 4018, 4072, 4156, 4180, 4244,
4248, 4292, 4376, 4400, 4464, 4468, 4512, 4596, 4620, 4684,
4688, 4732, 4816, 4840, 4904, 4908, 4952, 5036, 5061, 5126,
5141, 5206, 5221, 5286, 5301, 5366, 5381, 5446, 5461, 5526,
5541, 5606, 5621, 5686, 5701, 5766, 5781, 5846, 5861, 5926,
5941, 6006, 6022, 6088, 6114, ... and from there we take the
yellow junction above until 10587 -- « I'm quite astonished
by the slow confluence » - **Maximilian**).

[**10,587**], (+05=) 592, (+75=) 667, (+26=) 693, ... and from
there we take the grey junction above.

Development of the "proof" for the first 3-comma-number above,
2676:

[**267,6**], 82, 150, 171, 172, 183, 204, 236, 278, 340, 423,

427, 461, 535, 550, 605, 611, 667, 683, 759, 796, 893, 961,
 1000, 1011, 1012, 1023, 1044, 1075, 1116, 1167, 1228, 1299,
 1380, 1471, 1472, 1483, 1504, 1535, 1576, 1627, 1688, 1759,
 1840, 1931, 1932, 1943, 1964, 1995, 2036, 2088, 2150, 2232,
 2234, 2256, 2298, 2360, 2442, 2444, 2466, 2508, 2570, 2652,
 2654 and 2676.

[26,76], 143, 204, ... (yellow junction)

[2,676], 702, 769, 796, ... (grey junction)

The smallest 4-comma number **Maximilian** has found is 16867 - and the smallest 5-comma is 111688: what a marvel - six digits, five possible cuts, no zero inside! Any taker for more tetra- and pentacommatiles?

Many thanks again to **Maximilian**! We can define now the "hypercomma numbers" - such numbers would come back to life no matter where the split (legally) occurs: you cut them in two, they arise, reborn anew like a phoenix. $S_{\text{hypercomma}}$ (or S_{phoenix}) starts like this:

$S_{\text{hypercomma}} = 10, 20, 30, 40, 50, 60, 70, 80, 90, 109, 806, 1023,$
 1044, 2005, 2676, 3066, 3602, 4051, 6053, 6246, 8011, 8349, 9427,
 10022, 10074, 10587, 13090, 15031, 16867, 20088, 20699, 21698, 23108,
 29986, 30091, 30306, 32226, 40022, 40099, 40153, 41064, 41112, 50816,
 50841, 52010, ...

Best,
 É.

Comma numbers are now in the OEIS - start with [this](#) page.

A French post I've sent to a few friends yesterday (October 28th 2009):

Un coup de scie jamais n'abolira

Prenez le nombre 2676 et coupez-le en deux comme vous voulez
 -- par exemple ainsi : [267,6]

Nous allons construire une suite sur cette coupure et ajouter
 au moignon de droite (6) le nombre qui borde la plaie (ici 76).

La suite démarre comme ça : $S = 267,6,82$

D'où vient ce 82 ? Mais de la somme du moignon 6 et du nombre
 bordant la plaie, 76 -- on vient de vous le dire !

On... Horreur, en considérant S je vois apparaître une nou-
 velle plaie ! Et un nouveau moignon !

S = 267,6,82

Le moignon c'est 82 et la plaie c'est la nouvelle virgule -- laquelle est bordée par le nombre 68. Courage, fuyons (en avant), et ajoutons 68 à 82, pour voir (la somme fait 150) :

S = 267,6,82,150

Rrrrhââââ ! Nouveau moignon, nouvelle plaie, nouvelles lèvres formant une sorte d'emplâtre 21 ! On continue à suturer ! On ajoute à 150 ce 21 ! Scalpel, oxygène, bouge-toi Sophie, je ne vois rien !

S = 267,6,82,150,171

C'est plus une suite, c'est Frankenstein ! Tiens, les lèvres de la plaie forment à présent le "nombre" 01 -- c'est quoi ça ? Eh bien on disait que 01 ça valait 1, mon capitaine -- lequel 1 on ajoute à 171 et passez muscade ! Chirurgie de guerre, gamine, on ne finasse pas ! $171 + 1 = 172$, rompez !

S = 267,6,82,150,171,172

...

Vous avez compris, la suite peut continuer ainsi à l'infini -- nous l'arrêterons pourtant sur une révélation : 2676, le nombre que nous avons mutilé au départ (avec l'assentiment de la SPED -- Société Protectrice des Entiers Découpables), va réapparaître ! Regardez le travail, c'est du cousu main, c'est du Lacroix -- non, mieux, c'est du Lagerfeld !

S = 267, 6, 82, 150, 171, 172, 183, 204, 236, 278, 340, 423, 427, 461, 535, 550, 605, 611, 667, 683, 759, 796, 893, 961, 1000, 1011, 1012, 1023, 1044, 1075, 1116, 1167, 1228, 1299, 1380, 1471, 1472, 1483, 1504, 1535, 1576, 1627, 1688, 1759, 1840, 1931, 1932, 1943, 1964, 1995, 2036, 2088, 2150, 2232, 2234, 2256, 2298, 2360, 2442, 2444, 2466, 2508, 2570, 2652, 2654, 2676.

Olé ! Qu'il est mignon, le dernier moignon ! Son père tout craché ! Vive la médecine, Fleming, le protoxyde d'azote et le vaccin contre la grippe A !

Mais le meilleur est pour la fin : avec 2676 la coupe aurait pu avoir lieu n'importe où, [26,76] ou [2,676] au lieu de [267,6]: le nombre de départ aurait ressuscité de toute façon, tel un Phénix de ses bandes Velpeau !

Les nombres-phénix sont là -- et leur genèse :

<http://www.cetteadressecomportecinquantesignes.com/Commatile.htm>

à+,
É.