A152089
A76680
(1)

Date: Fri, 10 Sep 2010 13:29:15 -0600 (MDT)
From: Richard Guy <rkg@cpsc.ucalgary.ca>
To: Ed Pegg <ed@mathpuzzle.com>, math-fun <math-fun@mailman.xmission.com>
Cc: "Sequence Fans -- N. J. A. Sloane" <njas@research.att.com>, seqfan-bounces@list.seqfan.eu, Sequence Fanatics Discussion list <seqfan@list.seqfan.eu>
Subject: Re: [math-fun] Triangular+Triangular = Factorial

Just cleaning up old email, so this is a very,
very belated response, which may have been taken
care of by someone long ago. Copied to seqfans,
in case someone wants to make a sequence or
two out of it. Solutions of

x(x+1)/2 + y(y+1)/2 = z!

No solns for 3 =
6, 11, 12, 14, ...
A152089

are solutions of (2x+1)^2 + (2y+1)^2 = 8(z!) + 2
and the first few values of z for which there
are solutions can easily be ascertained:

(x,y,z) = (0,1,0), (0,1,1), (1,1,2), (0,3,3), (2,2,3),
(2,6,4), (0,15,5), (5,14,5), (45,89,7), (89,269,8),
(210,825,9), (760,2610,10), (1770,2030,10),
none for z = 11, 12, one for z = 13
(see Ed's solution below), none for z = 14,
two for z = 15 (see below), one for z = 16,
two for z = 17, none for z = 18, 19, 20,
two for z = 21, none for z = 22, 23,
two for z = 24, none for z = 25, 26,
eight for z = 27, one for z = 28,
two for z = 29, none for z = 30, 31,
four for z = 32, 33, sixteen for z = 34,
none for z = 35, 36, ..., 41,
two for z = 42, none for z = 43, 44, ..., 48,
sixteen for z = 49, none for z = 50, 51, 52, 53,
one for z = 54, none for z = 55, 56, ..., 65,
two for z = 66, none for z = 67,
sixteen for z = 68, ...
(E&OE, and PARI is slowing down a bit now: AND it would
take rather longer to find the actual solutions!) R.

On Tue, 20 Jan 2009, Ed Pegg
Jr wrote:

> A very belated response.
>
> Not squares, but related. Using Triangular numbers.
> 1 3 6 10 15 21
>
> T[3] = 3!
> T[3] + T[6] = 4!
> T[14]+T[5] = T[15] = 5!
> T[45] + T[89] = 7!
> T[210] + T[825] = 9!
> T[1770] + T[2030] = 10!
> T[71504] + T[85680] = 13!
> T[213384] + T[1603064] = T[299894] + T[1589154] = 15!
>
> I don't see an easy way to extend these. The density of triangular numbers seems to be sufficient for extended solutions.
>
> --Ed Pegg Jr
>
> Date: Mon, 3 Jul 2006 11:44:20 -0600 (MDT)

> From: Richard Guy <rkg@cpsc.ucalgary.ca>
> Reply-To: math-fun <math-fun@mailman.xmission.com>
> To: Number Theory List <NMBRTHRY@listserv.nodak.edu>,
>   Math Fun <math-fun@mailman.xmission.com>
> Subject: [math-fun] Factorial n
>
> Presumably 0! = 1! = 0^2 + 1^2.
>   2! = 1^2 + 1^2
>   6! = 12^2 + 24^2
>
> are the only integer solutions of
>
>   n! = x^2 + y^2
>
> but is there a proof? [Later:
for n > 6, n! will always contain a
prime factor of shape 4k - 1 raised to an
odd power, in fact raised to the first
power, by a suitable form of the prime
number theorem.] R.
> ____
> math-fun mailing list
> math-fun@mailman.xmission.com
> http://mailman.xmission.com/cgi-bin/mailman/listinfo/math-fun

---1618471419-318461834-1284070718=:30565--

Very interesting, Richard -- and Ed !

This suggests a few questions to me,
possible hard ones:

1) Can we prove there are infinitely many
solutions to

    T[x] + T[y] = z!

?

How about a probabilistic heuristic?

2) Generally, given reasonably simple functions

    F,G: Z+ -> Z+

when can we prove there are infinitely many
solutions to

    F(x) + F(y) = G(z)

?

Likewise, what about a probabilistic "proof" ?

--Dan

Date: Fri, 10 Sep 2010 17:30:29 -0600 (MDT)
From: Richard Guy <rkg@cpsc.ucalgary.ca>

What it boils down to is:

Are there infinitely many 4*n! + 1 which only
have factors of shape 4k + 1 (or possibly
some factors 4k + 3, but to an even power).

I'm inclined to think ''yes'', but this is
far beyond our reach. It may even be true
that there are infinitely many primes
4*n! + 1 ???? R.

From: Douglas McNeil <mcneil@hku.hk>

> probably relatively moderate amount of elliptic curve factoring will
> pick up the low hanging fruit.

Seemed straightforward enough. I can now exclude z values of

excluded: [6, 11, 12, 14, 18, 19, 20, 22, 23, 25, 26, 30, 31, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 67, 70, 71, 73, 75, 76, 77, 78, 82, 83, 84, 87, 88, 89, 90, 91, 92, 94, 97, 98, 101, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 115, 116, 117, 119, 120, 121, 122, 123, 124, 127, 128, 130, 132, 136, 137, 138, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 166, 167, 168, 169, 170, 172, 174, 175, 176, 177, 179, 180, 181, 182, 183, 185, 186, 187, 188, 191, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 213, 214, 215, 216, 218, 223, 225, 226, 227, 228, 229, 230, 231, 232, 236, 237, 238, 239, 240, 241, 243, 245, 246, 247, 249, 250, 251, 253, 255, 256, 257, 258, 260, 261, 262, 263, 265, 266, 267, 269, 270, 271, 272, 273, 274, 276, 278, 279, 281, 282, 283, 284, 285, 287, 288, 289, 290, 292, 293, 295, 296, 297, 298, 300, 301, 302, 303, 304, 306, 307, 309, 311, 313, 314, 315, 316, 317, 319, 320, 321, 322, 324, 327, 331, 333, 334, 335, 336, 338, 339, 340, 343, 344, 346, 348, 350, 352, 353, 354, 355, 356, 358, 359, 360, 362, 365, 366, 367, 368, 369, 370, 372, 373, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 394, 395, 398, 399, 400, 401, 403, 404, 405, 406, 407, 408, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 429, 432, 433, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 449, 450, 451, 452, 454, 456, 458, 459, 460, 462, 464, 469, 470, 472, 474, 479, 486, 487, 488, 489]

and can fully factor 8*(z!)+2 for

fully factored: [1, 2, 3, 4, 5, 7, 8, 9, 10, 13, 15, 16, 17, 21, 24, 27, 28, 29, 32, 33, 34, 42, 49, 54, 59, 66, 68, 72, 79, 85, 86, 95, 96, 102, 118, 129, 135, 164, 184, 190, 219, 221, 264, 351, 357, 457, 466]

This leaves only 7 numbers, [69, 74, 80, 81, 93, 99, 100] <= 100 as undecided (123 in total < 500) but they could be within reach. I set a pretty short time limit on the ecm runs, to get a first pass done overnight. Anyway, it should be complete up to z <= 68.

Note that there are two differences with results in this thread.
Trivial: 2 I think was put in by accident in the previous exclusion list (8*(2!)+2 = 18 = 9+9); I agree with everything else.
Non-trivial: the results quoted by R.K. Guy listed no solution for z=59, but 8*(59!)+2 has prime factors

[2, 8249057, 2174500369, 67820061593, 114446404287889, 11247609217977437, 354241345536913447681853]

in which all factors 3 mod 4 have even exponent (namely, 0).

Doug

[2, 8249057, 2174500369, 67820061593, 114446404287889,
11247609217977437, 354241345536913447681853]

in which all factors 3 mod 4 have even exponent (namely, 0).

Doug

--
Department of Earth Sciences
University of Hong Kong

Date: Mon, 13 Sep 2010 12:17:10 +0300
From: Georgi Guninski <guninski@guninski.com>

On Mon, Sep 13, 2010 at 09:35:14AM +0800, Douglas McNeil wrote:
> and can fully factor 8*(z!)+2 for
>
> fully factored: [1, 2, 3, 4, 5, 7, 8, 9, 10, 13, 15, 16, 17, 21, 24,
> 27, 28, 29, 32, 33, 34, 42, 49, 54, 59, 66, 68, 72, 79, 85, 86, 95,
> 96, 102, 118, 129, 135, 164, 184, 190, 219, 221, 264, 351, 357, 457,
> 466]
>
> This leaves only 7 numbers, [69, 74, 80, 81, 93, 99, 100] <= 100 as
> undecided (123 in total < 500) but they could be within reach. I set
> a pretty short time limit on the ecm runs, to get a first pass done
> overnight. Anyway, it should be complete up to z <= 68.
>
> Note that there are two differences with results in this thread.
> Trivial: 2 I think was put in by accident in the previous exclusion
> list (8*(2!)+2 = 18 = 9+9); I agree with everything else.
> Non-trivial: the results quoted by R.K. Guy listed no solution for
> z=59, but 8*(59!)+2 has prime factors
>

thanks for catching the multiplicity bug with 2, hopefully fixed now.

i can add 139 as fully factored solution (factorization at end):

and 294,286 as no solutions.

here are my results for z<=300

42 solutions:

2,3,4,5,7,8,9,10,13,15,16,17,21,24,27,28,29,32,33,34,42,49,54,59,66,72,79,85,86,95,96,1
02,118,129,135,139,164,184,190,219,221,264

52 with unknown status:

61, 68, 69, 71, 74, 80, 81, 93, 99, 100, 107, 109, 114, 125, 126, 131, 133, 134, 140, 1
41, 143, 162, 165, 171, 173, 178, 189, 192, 199, 208, 209, 212, 217, 220, 222, 224, 233
, 234, 235, 242, 244, 248, 252, 254, 259, 268, 275, 277, 280, 291, 296, 299

and the rest are without solutions.

took about 6 hours with opportunistic EC factoring.

all known to me <=300 (E&OE) , format is |SOLUTION(+NUMBER OF PRIME FACTORS)|

2(+2),3(+2),4(+2),5(+3),6(-),7(+2),8(+2),9(+2),10(+3),11(-),12(-),13(+2),14(-),15(+3),1
6(+2),17(+3),18(-),19(-),20(-),21(+3),22(-),23(-),24(+3),25(-),26(-),27(+5),28(+2),29(+
3),30(-),31(-),32(+4),33(+4),34(+6),35(-),36(-),37(-),38(-),39(-),40(-),41(-),42(+3),43
(-),44(-),45(-),46(-),47(-),48(-),49(+6),50(-),51(-),52(-),53(-),54(+2),55(-),56(-),57(
-),58(-),59(+7),60(-),62(-),63(-),64(-),65(-),66(+3),67(-),70(-),72(+3),73(-),75(-),76(
-),77(-),78(-),79(+6),82(-),83(-),84(-),85(+3),86(+2),87(-),88(-),89(-),90(-),91(-),92(
-),94(-),95(+4),96(+3),97(-),98(-),101(-),102(+5),103(-),104(-),105(-),106(-),108(-),11
0(-),111(-),112(-),113(-),115(-),116(-),117(-),118(+6),119(-),120(-),121(-),122(-),123(
-),124(-),127(-),128(-),129(+2),130(-),132(-),135(+5),136(-),137(-),138(-),139(+4),142(
-),144(-),145(-),146(-),147(-),148(-),149(-),150(-),151(-),152(-),153(-),154(-),155(-),
156(-),157(-),158(-),159(-),160(-),161(-),163(-),164(+4),166(-),167(-),168(-),169(-),17
0(-),172(-),174(-),175(-),176(-),
177(-),179(-),180(-),181(-),182(-),183(-),184(+5),185(-),186(-),187(-),188(-),190(+2),
191(-),193(-),194(-),195(-),196(-),197(-),198(-),200(-),201(-),202(-),203(-),204(-),205
(-),206(-),207(-),210(-),211(-),213(-),214(-),215(-),216(-),218(-),219(+5),221(+3),223(
-),225(-),226(-),227(-),228(-),229(-),230(-),231(-),232(-),236(-),237(-),238(-),239(-),
240(-),241(-),243(-),245(-),246(-),247(-),249(-),250(-),251(-),253(-),255(-),256(-),257
(-),258(-),260(-),261(-),262(-),263(-),264(+4),265(-),266(-),267(-),269(-),270(-),271(-
),272(-),273(-),274(-),276(-),278(-),279(-),281(-),282(-),283(-),284(-),285(-),286(-),2
87(-),288(-),289(-),290(-),292(-),293(-),294(-),295(-),297(-),298(-),300(-)

139+(4): [2, 288245637707785632653, 16377513261243153166696673, 81476289222097686957291
918164413236312063831684966837348397616965036974678013728645423662413401987536260023582
443949960880989765406242254362489665549605338264289123236751948012164984705741862629]

if someone is interested in the (partial) factorizations, let me know.

Seqfan Mailing list - http://list.seqfan.eu/

From: rcs@xmission.com
To: math-fun@mailman.xmission.com

Using typical probability arguments, there are infinitely many primes K = 4 N! + 1, but they are pretty thin:

Ignoring what we know about no-small-factors, the likelihood that 4 N! + 1 is prime is 1/log K, very roughly 1/N log N. The sum of this is about loglog N. By this calculation, we get a new prime every time N is raised to the power e. The numbers K are very thin indeed.

We can correct for the fact that our number shape has no divisors <=N (in fact, N+1 and N+2 are also impossible for N>3) by bumping up the likelihood by prod(P/(P-1)) for P<=N (and P prime of course). IIRC this is O(logN), making our summand O(1/N), and the sum O(logN). This is comparable to Mersenne primes in sparsity for K, and MP exponents for sparsity in N. I.e., we get a new prime every time N doubles (and K squares).

The likelihood that a number K of this shape is the
sum of two squares but not necessarily prime,
(again IIRC) is O(1/sqrt log K). The approximate
sum of 1/sqrt(N logN) seems to be O(sqrt(N/logN)),
indicating quite a few N are winners.

[No, I can't prove any of this.]

Rich

Date: Tue, 14 Sep 2010 10:11:09 -0600 (MDT)
From: Richard Guy <rkg@cpsc.ucalgary.ca>

Many thanks for all that help. Just to clear up
one or two things that might be in doubt:

1. I omitted the solution (x,y,z) = (89,269,8).

2. For z = 59, there are 32 solutions, which
some hero(ine) may like to calculate from

```
? factor(8*59!+2)
%64 =
[2 1]

[8249057 1]

[2174500369 1]

[67820061593 1]

[114446404287889 1]

[11247609217977437 1]

[35424134553691344768l853 1]
```

3. There are no solutions for z = 69, as may
be seen from

```
? factor(4*69!+1)
%74 =
[96493309986243088621721365030167853723206604771 1]

[709365042830587908924013090550258863264755833052513l 1]
```

4. The first values of z about which I am ignorant
are z = 71, 74, 80, 81, 93, 99, 100, ...

5. If one is willing to accept probable primes,
then the following sequence of z for which
4 * z! + 1 is prime may be complete for
z < 300 or more?? --- comments?

0, 1, 4, 7, 8, 9, 13, 16, 28, 54, 129, 190, ...
(A076680)

Date: Tue, 14 Sep 2010 10:17:27 -0600 (MDT)
From: Richard Guy <rkg@cpsc.ucalgary.ca>

7

Sorry, I missed 86 from that last sequence. R.

Date: Tue, 14 Sep 2010 19:57:26 +0300
From: Georgi Guninski <guninski@guninski.com>

On Tue, Sep 14, 2010 at 10:11:09AM -0600, Richard Guy wrote:
>
> 4. The first values of z about which I am ignorant
> are z = 71, 74, 80, 81, 93, 99, 100, ...
>

attaching my and some of Douglas McNeil's factorizations.

summary, check attached file for factorizations (gzipped due to list, limitations, also available at:
http://stefan.guninski.com/seq/guyres2.txt):

71 is not a solution (Douglas McNeil)
74 is not a solution
80 solution, 3 factors
81 solution, 4 factors
93 - unknown to me, trying it with EC
99 solution, 6 factors
100 is not a solution

the rest go:
101(-),102(+5),103(-),104(-),105(-),106(-),108(-),110(-),111(-),112(-),113(-),115(-),116(-),117(-),118(+6),119(-),120(-),121(-),122(-),123(-),124(-),127(-),128(-),129(+2)... more, check attached factors

Date: Thu, 16 Sep 2010 13:22:27 +0700
From: Warut Roonguthai <warut822@gmail.com>

I've added the factors of numbers of the form 4*z!+1 that I recently found to the factor database:

http://factordb.com/index.php?query=4*z%21%2B1

In particular, 4*69!+1 =
96493309986243088621721365030167853723206604771 * P52, so there is no solution for z = 69.

Feel free to use them to compute related sequences for OEIS (enough for z up to 80). I'll continue factoring as my computer time permits.

Warut

Date: Thu, 16 Sep 2010 10:29:10 +0300
From: Georgi Guninski <guninski@guninski.com>

http://oeis.org/classic/A076680
Numbers n such that 4*n! + 1 is a prime.
0, 1, 4, 7, 8, 9, 13, 16, 28, 54, 86, 129, 190, 351, 466, 697, 938, 1510, 3396, 4057, 4384

>From seqfan-bounces@list.seqfan.eu Sat Sep 11 14:55:56 2010
Received: from mail-black.research.att.com (mail-black.research.att.com [135.207.127.54]) by unixmail.research.att.com (8.13.7+Sun/8.13.7) with ESMTP id o8BItuqe012194 for <njas@unixmail.research.att.com>; Sat, 11 Sep 2010 14:55:56 -0400 (EDT)

From G. Guninski Sep 11 2010 (8)


```
> and the first few values of  z for which there
> are solutions can easily be ascertained:
>
> (x,y,z) = (0,1,0), (0,1,1), (1,1,2), (0,3,3), (2,2,3),
> (2,6,4), (0,15,5), (5,14,5), (45,89,7), (89,269,8),
> (210,825,9), (760,2610,10), (1770,2030,10),
> none for  z = 11, 12, one for  z = 13
> (see Ed's solution below), none for  z = 14,
> two for  z = 15 (see below), one for  z = 16,
> two for  z = 17, none for  z = 18, 19, 20,
> two for  z = 21, none for  z = 22, 23,
> two for  z = 24, none for  z = 25, 26,
> eight for  z = 27, one for  z = 28,
> two for  z = 29, none for  z = 30, 31,
> four for  z = 32, 33, sixteen for  z = 34,
> none for  z = 35, 36, ..., 41,
> two for  z = 42, none for  z = 43, 44, ..., 48,
> sixteen for  z = 49, none for  z = 50, 51, 52, 53,
> one for  z = 54, none for  z = 55, 56, ..., 65,
> two for z = 66, none for  z = 67,
> sixteen for  z = 68, ...
> (E&OE, and PARI is slowing down a bit now: AND it would
> take rather longer to find the actual solutions!)    R.
>
```

based on partial factorization and condition for sum of two squares i get no solution f
or these up to 500:

2,6,11,12,14,18,19,20,22,23,25,26,30,36,37,39,43,44,45,46,47,51,52,57,58,62,64,67,75,82
,84,88,89,90,97,98,101,105,106,112,113,115,116,117,121,123,124,127,132,136,137,138,142,
144,146,148,150,151,153,155,156,157,158,159,161,163,167,170,172,174,175,182,185,186,188
,195,196,207,210,211,215,225,226,227,228,231,232,236,237,249,251,256,257,258,261,262,26
3,265,266,269,270,274,278,279,282,284,285,287,288,289,292,293,295,297,298,300,301,307,3
11,319,320,324,331,333,334,338,339,343,346,354,355,356,359,360,365,367,368,369,372,380,
381,384,387,388,390,394,395,398,399,400,401,404,405,406,414,420,421,423,424,425,427,432
,433,436,437,438,439,443,445,447,449,452,454,459,462,469,470,472,474,479,486,487,488,48
9

(not being in the list does not guarantee existence of solution).

out of pure luck the partial factorization gives efficient solution for
these z < 500:

72 85 86 129 190 351 457 466

8*(466!)+2 turns out to be semiprime with the trivial factor 2 and a solution is:

z= 466
x,y
8522292172914161718795957540506016669660975290159057309883412641651407777356567248305091178570348100667110021193361960094485157711459267831323704686581894534930283811471043553969771822214144791743093057811462222007440750869521161030637173616479545006466843987009376107675271111867729388859391942165682997300781086006132890033407624082753228734124641069761814680484584727788292639571694573923319472511409731419629675744455121809040164391071455566609038001971220156130142881830506065817385744083580521683436489157934604840535336434763563592249235379885099590217418325982003657784055098726909736010812269074964382891309557974805098011002443162479258419780001958074401355970170064153911182533553738010819908084710120606360788655042121907713970212553601422325908197238686236324735214525384652000286001583099728928825825982905325063374862449143276229081054090978323216472969295996118815854608437718947144356973559886285411827854764935121573220769145897689240103541864401621974084254024250040889654542416397981132732227672491066205502593667801446132 9

ok?= True

list.seqfan.eu; Sat, 11 Sep 2010 23:26:00 +0200
Received: by cannabis.dataforce.net (Postfix, from userid 12794) id 25C0217EAB; Sun, 1
2 Sep 2010 01:25:59 +0400 (MSD)
Date: Sun, 12 Sep 2010 00:25:59 +0300
From: Georgi Guninski <guninski@guninski.com>
To: Sequence Fanatics Discussion list <seqfan@list.seqfan.eu>
Message-Id: <20100911212559.GC1821@sivokote.iziade.m$>
References: <949728.32098.qm@web30805.mail.mud.yahoo.com> <alpine.LRH.2.00.10090915113
60.30565@csl> <20100911185543.GA1821@sivokote.iziade.m$> <alpine.LRH.2.00.10091113333
10.15565@csl>
Mime-Version: 1.0
Content-Disposition: inline
In-Reply-To: <alpine.LRH.2.00.1009111333310.15565@csl>
Header: best read with a sniffer
User-Agent: Mutt/1.5.20 (2009-06-14)
Subject: [seqfan] Re: [math-fun] Triangular+Triangular = Factorial
X-Beenthere: seqfan@list.seqfan.eu
X-Mailman-Version: 2.1.9
Precedence: list
Reply-To: Sequence Fanatics Discussion list <seqfan@list.seqfan.eu>
List-Id: Sequence Fanatics Discussion list <seqfan.list.seqfan.eu>
List-Unsubscribe: <http://list.seqfan.eu/cgi-bin/mailman/listinfo/seqfan>, <mailto:seq
fan-request@list.seqfan.eu?subject=unsubscribe>
List-Archive: <http://list.seqfan.eu/pipermail/seqfan>
List-Post: <mailto:seqfan@list.seqfan.eu>
List-Help: <mailto:seqfan-request@list.seqfan.eu?subject=help>
List-Subscribe: <http://list.seqfan.eu/cgi-bin/mailman/listinfo/seqfan>, <mailto:seqfa
n-request@list.seqfan.eu?subject=subscribe>
Content-Type: text/plain; charset="us-ascii"
Content-Transfer-Encoding: 7bit
Sender: seqfan-bounces@list.seqfan.eu
Errors-To: seqfan-bounces@list.seqfan.eu
Status: RO

On Sat, Sep 11, 2010 at 01:36:21PM -0600, Richard Guy wrote:
> Well done! Now can someone show that there
> ARE solutions for the other values of z ?? R.
>

for z<2500 and not in A076680 another nontrivial solution is z=546.

probably relatively moderate amount of elliptic curve factoring will
pick up the low hanging fruit.
