

Wolfdieter Lang, corrected version Jul 30 2013  
 (the order of the partitions was incorrect for n = 11 .. 15)

a(n,k) tabf head (staircase) for A117506 W. Lang, Apr 13, 2006  
 Dimensions of S\_n irreps, equivalently nr. of Young tableaux for the Young  
 diagram corresponding to a partitions of n, written in Abramowitz-Stegun order.

n\k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	1																													
2	1	1																												
3	1	2	1																											
4	1	3	2	3	1																									
5	1	4	5	6	5	4	1																							
6	1	5	9	5	10	16	5	10	9	5	1																			
7	1	6	14	14	15	35	21	21	20	35	14	15	14	6	1															
8	1	7	20	28	14	21	64	70	56	42	35	90	56	70	14	35	64	28	21	20	7	1								
9	1	8	27	48	42	28	105	162	84	120	168	42	56	189	216	216	168	84	70	189	120	162	42	56	105	48	28	27	8	1
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n\k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30

The sequence of row lengths is A000041: [1, 2, 3, 5, 7, 11, 15, 22, 30, 42,...] (partition numbers).

For the ordering of this tabf array a(n,k) see Abramowitz-Stegun ref. pp. 831-2.

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E.g. a(4,4) refers to the fourth partition of n=4 in this ordering, namely (1^2,2)=[1,1,2], but here written with falling part numbers as

[2,1,1] corresponding to the Young diagram  $\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \\ \hline \square & \\ \hline \end{array}$ . The associated hook length diagram is  $\begin{array}{cc} 4 & 1 \\ 2 & \\ 1 & \end{array}$ , and  $a(4,4) = 4! / (4 \cdot 1 \cdot 2 \cdot 1) = 3$

See the theorem on the bottom of p.9 of the Wybourne reference for this alternative way to compute the dimension.

The next rows are:

n=10: (42 partitions)  
 [1, 9, 35, 75, 90, 42, 36, 160, 315, 288, 225, 450, 252, 210, 84, 350, 567, 300, 525, 768, 210, 300, 252, 126, 448, 525, 567, 450, 288, 42, 126, 350, 225, 315, 90, 84, 160, 75, 36, 35, 9, 1];

n=11: (56 partitions)  
 [1, 10, 44, 110, 165, 132, 45, 231, 550, 693, 330, 385, 990, 990, 660, 462, 120, 594, 1232, 1155, 1100, 2310, 1320, 1188, 825, 1320, 462, 210, 924, 1540, 825, 1540, 2310, 660, 1155, 990, 330, 252, 924, 1100, 1232, 990, 693, 132, 210, 594, 385, 550, 165, 120, 231, 110, 45, 44, 10, 1];

n=12: (77 partitions)

[1, 11, 54, 154, 275, 297, 132, 55, 320, 891, 1408, 1155, 616, 1925, 2673, 1320, 1650, 2112, 462, 165, 945, 2376, 3080, 1485, 2079, 5632, 5775, 4158, 2970, 1925, 4455, 2640, 2970, 462, 330, 1728, 3696, 3520, 3564, 7700, 4455, 4158, 3520, 5775, 2112, 1485, 1320, 462, 2100, 3564, 1925, 3696, 5632, 1650, 3080, 2673, 1155, 132, 462, 1728, 2079, 2376, 1925, 1408, 297, 330, 945, 616, 891, 275, 165, 320, 154, 55, 54, 11, 1];

n=13: (101 partitions)

[1, 12, 65, 208, 429, 572, 429, 66, 429, 1365, 2574, 2860, 1287, 936, 3432, 6006, 5148, 3575, 6435, 3432, 2574, 220, 1430, 4212, 6864, 5720, 3640, 12012, 17160, 8580, 11440, 15015, 3432, 4004, 12012, 12870, 11583, 8580, 3432, 495, 3003, 7800, 10296, 5005, 7371, 20592, 21450, 16016, 11583, 9009, 21450, 12870, 15015, 2574, 5005, 8580, 3432, 792, 4290, 9360, 9009, 9360, 20592, 12012, 11440, 10296, 17160, 6435, 5720, 5148, 1287, 924, 4290, 7371, 4004, 7800, 12012, 3575, 6864, 6006, 2860, 429, 792, 3003, 3640, 4212, 3432, 2574, 572, 495, 1430, 936, 1365, 429, 220, 429, 208, 66, 65, 12, 1];

n=14: (135 partitions)

[1, 13, 77, 273, 637, 1001, 1001, 429, 78, 560, 2002, 4368, 6006, 4576, 1365, 5733, 12012, 14014, 6435, 7007, 16016, 15015, 9009, 6006, 286, 2079, 7007, 13650, 15444, 7007, 6006, 23296, 42042, 36608, 27027, 50050, 27027, 21021, 7644, 28028, 42042, 21450, 35035, 48048, 12012, 15015, 12012, 715, 4928, 15015, 24960, 21021, 14014, 47775, 69498, 35035, 48048, 64064, 15015, 20384, 63063, 68640, 64064, 48048, 21021, 14014, 35035, 21450, 27027, 6006, 1287, 8085, 21450, 28665, 14014, 21021, 59904, 63063, 48048, 35035, 28665, 69498, 42042, 50050, 9009, 21021, 36608, 15015, 7007, 6435, 1716, 9504, 21021, 20384, 21450, 47775, 28028, 27027, 24960, 42042, 16016, 15444, 14014, 4576, 429, 1716, 8085, 14014, 7644, 15015, 23296, 7007, 13650, 12012, 6006, 1001, 1287, 4928, 6006, 7007, 5733, 4368, 1001, 715, 2079, 1365, 2002, 637, 286, 560, 273, 78, 77, 13, 1];

n=15: (176 partitions)

[1, 14, 90, 350, 910, 1638, 2002, 1430, 91, 715, 2835, 7007, 11375, 11583, 5005, 1925, 9100, 22113, 32032, 25025, 12740, 35035, 45045, 21450, 25025, 30030, 6006, 364, 2925, 11088, 25025, 35100, 27027, 9450, 42042, 91000, 108108, 50050, 57330, 135135, 128700, 80080, 54054, 13650, 58968, 112112, 100100, 90090, 175175, 96525, 81081, 50050, 75075, 24024, 1001, 7722, 26950, 53625, 61425, 28028, 24948, 100100, 184275, 162162, 122850, 231660, 126126, 100100, 42042, 159250, 243243, 125125, 210210, 292864, 75075, 100100, 81081, 34398, 112112, 125125, 126126, 96525, 54054, 6006, 2002, 14300, 44550, 75075, 63700, 43120, 150150, 221130, 112112, 156000, 210210, 50050, 70070, 221130, 243243, 231660, 175175, 80080, 63700, 162162, 100100, 128700, 30030, 28028, 50050, 21450, 3003, 19305, 51975, 70070, 34398, 51975, 150150, 159250, 122850, 90090, 75075, 184275, 112112, 135135, 25025, 61425, 108108, 45045, 27027, 25025, 5005, 3432, 19305, 43120, 42042, 44550, 100100, 58968, 57330, 53625, 91000, 35035, 35100, 32032, 11583, 1430, 3003, 14300, 24948, 13650, 26950, 42042, 12740, 25025, 22113, 11375, 2002, 2002, 7722, 9450, 11088, 9100, 7007, 1638, 1001, 2925, 1925, 2835, 910, 364, 715, 350, 91, 90, 14, 1].

etc.

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The row sums give A000085 (total number of Young tableaux with n cells):

[1, 2, 4, 10, 26, 76, 232, 764, 2620, 9496,...]

##### e.o.f. #####