SOME SEQUENCES ARISING AT THE INTERFACE OF REPRESENTATION THEORY AND HOMOTOPY THEORY

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1. What the numbers are

Let $V = \oplus_{k=1}^{\infty} V_k$ be the graded vector space $H_*(\mathbb{PC}^\infty)[1]$, which has Poincaré series $p(t) = \frac{t}{1-t^2}$. The q_m below are the dimensions of the free graded Lie algebra L on V.

There is a graded involution θ on V induced by an involution on $\mathbb{PC}^\infty$, which acts on V_{2k+1} as $(-1)^k$. The q_m^+ below are the dimensions of the $+1$-eigenspaces of θ on the graded components of L.

These sequences are inputs for the topological sequences which follow.

The algebraic varieties below all come in sequences $X_n, n = 1, 2, \ldots ; X_n$ has roughly n^2 Betti numbers, of which the first n are proven in my paper with Segal to be independent of n, i.e. $\beta_i(X_n) = \beta_i$ for $n \geq i$. The last 3 sequences are the stable values ("β_i") for the varieties described.

2. The numbers

The first fifty of the q_m $(m = 1, 2, \ldots)$ are given by

1, 1, 1, 1, 2, 3, 4, 5, 8, 13, 18, 25, 40, 62, 90, 135, 210, 324, 492, 750,
1164, 1809, 2786, 4305, 6710, 10460, 16264, 25350, 39650, 62057, 97108, 152145,
238818, 375165, 589520, 927200, 1459960, 2300346, 3626200, 5720274, 9030450,
14264242, 22542396, 35644500, 56393760, 89264833, 141358274, 223955235,
354975428, 562878415, 892893050

Similarly, the first fifty of the q_m^+ $(m = 1, 2, \ldots)$ are given by

1, 1, 0, 0, 1, 2, 2, 2, 4, 7, 9, 12, 20, 32, 45, 66, 105, 164, 246, 372, 582,
909, 1393, 2146, 3355, 5240, 8132, 12660, 19825, 31051, 48554, 76038, 119409,
187635, 294760, 463520, 729980, 1150296, 1813100, 2859948, 4515225, 7132412,
11271198, 17821800, 28196880, 44633113, 70679137, 111976538, 177487714,
281440885, 446446525

For the various Poincaré series $\sum_{i=0}^{\infty} a_i t^i$, we list the coefficients in the order $a_0, a_1, a_2, \ldots$.

The stable Poincaré series for the Lie algebra of type A, i.e. the variety of complex $n \times n$ matrices with distinct eigenvalues, has coefficients

$1, 1, 0, 1, 2, 4, 6, 9, 17, 30, 47, 75, 131, 221, 358, 589,$
$987, 1640, 2695, 4435, 7346, 12153, 19993, 32886, 54232, 89349,$
$146880, 241416, 396995, 652509, 1071496,$
$1758884, 2887206, 4737820, 7770521, 12740561, 20886071, 34229894,$
$56079853, 91853755, 150417991, 246264599, 403083757, 659619347, 1079213964,$
$1765368168, 2887185977, 4720983277, 7718168486, 12615970769, 20618279891,$
32274001177

The stable Poincaré series for the Lie group of type A, i.e. the variety of complex *non-singular* $n \times n$ matrices with distinct eigenvalues, has coefficients

$1, 2, 2, 4, 8, 15, 27, 47, 85, 153, 268, 466, 818, 1430, 2475, 4273, 7377,$
$12701, 21786, 37282, 63719, 108719, 185085, 314537, 533861, 904861,$
$1531370, 2588298, 4369783, 7369174, 12413458, 20889007, 35118175, 58986118,$
$98987608, 165976467, 278079625, 465545425, 778818832, 1301988118,$
$2175145088, 3631544050, 6059354904, 10104253702, 16839738813, 28049741336,$
$46697452162, 77702808607, 129231243278, 214829039377, 356960394060$

The stable Poincaré series for the Lie algebra of type B or C has coefficients

$1, 2, 2, 2, 2, 5, 12, 20, 29, 45, 75, 127, 213, 349, 568, 928, 1520, 2495,$
$4094, 6699, 10950, 17908, 29286, 47864, 78196, 127722, 208562, 340463, 555623,$
$906581, 1478943, 2412061, 3933043, 6412032, 10451715, 17033326, 27754758,$
$45217744, 73657343, 119966288, 195362519, 318101973, 517889138, 843050542,$
$1372204935, 2233240667, 3634168102, 5913284785, 9620742031, 15651144529,$
$25459088704, 40254595697$