OEIS SEQUENCE A088750

$a(n)$ = number of the zero of the Riemann zeta-function on the same line as the Gram point g_{n-2}. It is only well-defined if the Riemann hypothesis is true.

In the figures we represent the region $(-1,5)\times(0,200)$ of the complex plane. The thin curved lines are those points where $\Re\zeta(s)=0$, so at these points $\zeta(s)$ is purely imaginary. The thick curved lines are those points where $\zeta(s)$ is real. We call it the imaginary and real lines of $\zeta(s)$. The zeros of $\zeta(s)$ are the points $\frac{1}{2}+i\gamma_n$ where the imaginary and real curves cut. We can see these γ_n in the figure. The other points g_n are the Gram points. Usually we have

$$g_{n-2}<\gamma_n<g_{n-1}, \qquad \text{not always true.}$$

It is frequent that a real line contains a zero and a Gram point. For example $\frac{1}{2}+i\gamma_6$ is in the same real line that $\frac{1}{2}+ig_5$. According to the definition this means that $a(7)=6$.

There are some lines, I call it parallel, cross the represented section and contains alternatively a zero or a Gram point. It is natural to consider that the parallel line that contains a Gram point continues with the next parallel line above it that contains a zero. For example the line marked 53 continues with the line marked 59. So we consider that g_{13} and γ_{16} are on the same line.

We have in the same line:

$$\{\gamma_1,g_{-1}\},\{\gamma_2,g_0\},\{\gamma_3,g_1\},\{\gamma_4,g_2\},\{\gamma_5,g_3\},\{\gamma_7,g_4\},\{\gamma_6,g_5\},\{\gamma_8,g_6\},\{\gamma_{10},g_7\},\{\gamma_9,g_8\},\dots$$

Hence the sequence start with (order the g_n and take the subindex of the corresponding γ's)

$$1,2,3,4,5,7,6,8,10,9,\dots$$

Cuadro 1. Values of A088750

n	$a(n)$	n	$a(n)$	n	$a(n)$	n	$a(n)$	n	$a(n)$	n	$a(n)$	n	$a(n)$	n	$a(n)$
1	1	11	11	21	21	31	30	41	41	51	53	61	63	71	70
2	2	12	13	22	24	32	31	42	44	52	51	62	61	72	71
3	3	13	12	23	22	33	33	43	42	53	52	63	62	73	73
4	4	14	14	24	23	34	35	44	43	54	54	64	64	74	74
5	5	15	16	25	25	35	34	45	45	55	55	65	67	75	78
6	7	16	15	26	27	36	36	46	46	56	57	66	65	76	75
7	6	17	17	27	26	37	37	47	48	57	56	67	66	77	76
8	8	18	18	28	28	38	40	48	47	58	58	68	68	78	77
9	10	19	20	29	29	39	38	49	49	59	59	69	69	79	79
10	9	20	19	30	32	40	39	50	50	60	60	70	72	80	

We may chek all these values in the graphs, for example $a(78) = 77$ means that γ_{77} is on the same real line as the Gram point g_{76}