Cycle indices of linear group, linear self-dual group, affine group and affine self-dual group

Explanation:
For linear group GL(n,2) the first column multiply only first addends without coefficients from the third column;
for linear self-dual group LSD(n,2) the second column multiply only first addends without coefficients from the third column;
for affine group AG(n,2) the first column multiply all addends (with coefficients) from the third column;
for affine self-dual group M(n,2) the second column multiply all addends (with coefficients) from the third column.

Exempla:

$$Z(GL(2,2);x_1,x_2,...)=\frac{1}{6}(\ x_1^4+3x_1^2x_2+2x_1x_3);$$

$$Z(AG(2,2);x_1,x_2,...)=\frac{1}{24}(\ (x_1^4+3x_2^2)+3*(2x_1^2x_2+2x_4)+2*(4x_1x_3))=\frac{1}{24}(\ x_1^4+3x_2^2+6x_1^2x_2+6x_4+8x_1x_3);$$

$$Z(LSD(2,2);x_1,x_2,...)=\frac{1}{2}(x_1^4+x_1^2x_2);$$

$$Z(M(2,2);x_1,x_2,...)=\frac{1}{8}((\ x_1^4+3x_2^2)+(2x_1^2x_2+2x_4))=\frac{1}{8}(\ x_1^4+3x_2^2+2x_1^2x_2+2x_4).$$

GL(n,2)	LSD(n,2)	AG(n,2)	M(n,2)	
1	1	$x_1^4+3x_2^2$		$n=2$
3	1	$2x_1^2x_2+2x_4$		
2	0	$4x_1x_3$		
6	2	24	8	

1	1	$x_1^8+7x_2^4$		$n=3$
21	9	$2x_1^4x_2^2+2x_2^4+4x_4^2$		
42	6	$4x_1^2x_2x_4+4x_4^2$		
56	8	$4x_1^2x_3^2+4x_2x_6$		
2*24	2*0	$8x_1x_7$		
168	24	1344	192	

1	1	$x_1^{16}+15x_2^8$	$n=4$
105	49	$2x_1^8x_2^4+6x_2^8+8x_4^4$	
210	42	$4x_1^4x_2^6+12x_4^4$	
1260	252	$4x_1^4x_2^2x_4^2+4x_2^4x_4^2+8x_4^4$	
2520	168	$8x_1^2x_2x_4^3+8x_8^2$	
1120	224	$4x_1^4x_3^4+12x_2^2x_6^2$	
3360	224	$8x_1^2x_2x_3^2x_6+8x_4x_{12}$	
112	0	$16x_1x_3^5$	
1680	0	$16x_1x_3x_6^2$	
2*2880	2*192	$8x_1^2x_7^2+8x_2x_{14}$	
2*1344	2*0	$16x_1x_{15}$	
1344	0	$16x_1x_5^3$	
20160	1344	322560 21504	

1	1	$x_1^{32}+31x_2^{16}$	$n=5$
465	225	$2x_1^{16}x_2^8+14x_2^{16}+16x_4^8$	
6510	1470	$4x_1^8x_2^{12}+4x_2^{16}+24x_4^8$	
26040	5880	$4x_1^8x_2^4x_4^4+12x_2^8x_4^4+16x_4^8$	
78120	7560	$8x_1^4x_2^6x_4^4+24x_4^8$	
312480	30240	$8x_1^4x_2^2x_4^6+8x_2^4x_4^6+16x_8^4$	
624960	20160	$16x_1^2x_2x_4^3x_8^2+16x_8^4$	
19840	4480	$4x_1^8x_3^8+28x_2^4x_6^4$	
416640	40320	$8x_1^4x_2^2x_3^4x_6^2+8x_2^4x_6^4+16x_4^2x_{12}^2$	
833280	26880	$16x_1^2x_2x_3^2x_4x_6x_{12}+16x_4^2x_{12}^2$	
55552	1792	$16x_1^2x_3^{10}+16x_2x_6^5$	
833280	26880	$16x_1^2x_3^2x_6^4+16x_2x_6^5$	
2*238080	2*23040	$8x_1^4x_7^4+24x_2^2x_{14}^2$	
2*714240	2*23040	$16x_1^2x_2x_7^2x_{14}+16x_4x_{28}$	
2*476160	2*0	$32x_1x_3x_7x_{21}$	
2*666624	2*21504	$16x_1^2x_{15}^2+16x_2x_{30}$	
666624	21504	$16x_1^2x_5^6+16x_2x_{10}^3$	
6*322560	0	$32x_1x_{31}$	
9999360	322560	319979520 10321920	

GL(6,2)	LSD(6,2)	AG(6,2)	M(6,2)
1	1	$x_1^{64}+63x_2^{32}$	
1953	961	$2x_1^{32}x_2^{16}+30x_2^{32}+32x_4^{16}$	
136710	32550	$4x_1^{16}x_2^{24}+12x_2^{32}+48x_4^{16}$	
234360	26040	$8x_1^8x_2^{28}+56x_4^{16}$	
468720	111600	$4x_1^{16}x_2^8x_4^8+28x_2^{16}x_4^8+32x_4^{16}$	
9843120	1093680	$8x_1^8x_2^{12}x_4^8+8x_2^{16}x_4^8+48x_4^{16}$	
13124160	624960	$16x_1^4x_2^6x_4^{12}+48x_4^{16}$	
26248320	2916480	$8x_1^8x_2^4x_4^{12}+24x_2^8x_4^{12}+32x_8^8$	
78744960	3749760	$16x_1^4x_2^6x_4^{12}+16x_4^{16}+32x_8^8$	
314979840	14999040	$16x_1^4x_2^2x_4^6x_8^4+16x_2^4x_4^6x_8^4+32x_8^8$	
629959680	9999360	$32x_1^2x_2x_4^3x_8^6+32x_8^8$	
333312	79360	$4x_1^{16}x_3^{16}+60x_2^8x_6^8$	
34997760	3888640	$8x_1^8x_2^4x_3^8x_6^4+24x_2^8x_6^8+32x_4^4x_{12}^4$	
69995520	3333120	$16x_1^4x_2^6x_3^4x_6^6+48x_4^4x_{12}^4$	
419973120	19998720	$16x_1^4x_2^2x_3^4x_4^2x_6^2x_{12}^2+16x_2^4x_4^2x_6^4x_{12}^2+32x_4^4x_{12}^4$	
839946240	13332480	$32x_1^2x_2x_3^2x_4^3x_6x_{12}^3+32x_8^2x_{24}^2$	
18665472	888832	$16x_1^4x_3^{20}+48x_2^2x_6^{10}$	
279982080	13332480	$16x_1^4x_3^4x_6^8+48x_2^2x_6^{10}$	
55996416	888832	$32x_1^2x_2x_3^{10}x_6^5+32x_4x_{12}^5$	
839946240	13332480	$32x_1^2x_2x_3^2x_6^9+32x_4x_{12}^5$	
1111104	0	$64x_1x_3^{21}$	
34997760	0	$64x_1x_3^5x_6^8$	
419973120	0	$64x_1x_3x_6^2x_{12}^4$	
2*17141760	2*1904640	$8x_1^8x_7^8+56x_2^4x_{14}^4$	
2*359976960	2*17141760	$16x_1^4x_2^2x_7^4x_{14}^2+16x_2^4x_{14}^4+32x_4^2x_{28}^2$	
2*719953920	2*11427840	$32x_1^2x_2x_4x_7^2x_{14}x_{28}+32x_4^2x_{28}^2$	
2*959938560	2*152371120	$32x_1^2x_3^2x_7^2x_{21}^2+32x_2x_6x_{14}x_{42}$	
2*5713920	2*0	$64x_1x_7^9$	
411402240	0	$64x_1x_7^9$	
2*359976960	2*0	$64x_1x_7x_{14}^4$	
2*223985664	2*10665984	$16x_1^4x_{15}^4+48x_2^2x_{30}^2$	
2*671956992	2*10665984	$32x_1^2x_2x_{15}^2x_{30}+32x_4x_{60}$	
223985664	10665984	$16x_1^4x_5^{12}+48x_2^2x_{10}^6$	
671956992	10665984	$32x_1^2x_2x_5^6x_{10}^3+32x_4x_{20}^3$	
2*447971328	2*0	$64x_1x_3x_{15}^4$	
447971328	0	$64x_1x_3x_5^3x_{15}^3$	
6*650280960	6*10321920	$32x_1^2x_{31}^2+32x_2x_{62}$	
6*319979520	6*0	$64x_1x_{63}$	
2*319979520	2*0	$64x_1x_{21}^3$	
319979520	0	$64x_1x_9^7$	
20158709760	319979520	1290157424640	20478689280