Scan ~~[illegible]~~
A61396

Jon Aubrey
Letter to NJAS
~~1978~~ ~~1979~~
august 1980
3 ~~4~~ sides

Don't scan the cartoon

f91

N.J.A. Sloane
Bell Labs
600 Mountain Ave
Murray Hill, New Jersey 07974

RIFFS

~~A4117~~ A61396

800804

521 Sunrise Court
East Lansing, Mich.
48823

AWBREY

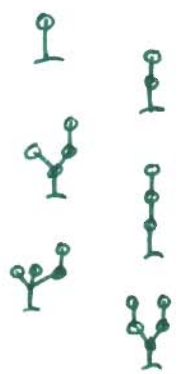
August 4 1980

Dear Mr. Sloane,

i don't know of any published references to the sequence 1, 2, 6, 20, 73, 281, 1124, 4488, ... or to anything resembling riffs, but there are a few items of context i can supply. unfortunately, i have just gone thru a change of address + can't find notes on exact references on anything. i haven't worked out the sequence any further, but am returning to the problem with some fancier methods + will let you know if i have any luck. the rest of this is less immediate.

riffs came out of an investigation with several heads: either 'number-theoretic codings of graphs', or 'graph-theoretic models of the integers', or 'measures of complexity on the integers.' from a musing together of Gödel numberings + the Zermelo-Von Neumann construction of integers, i was led to the following map of integers into planted plane trees, based on their multiplicative structure:

$1 = 2^0 = (\,) =$
$2 = 2^1 = ((\,)) =$
$3 = 2^0 3^1 = (\,)((\,)) =$
$4 = 2^2 = (((\,))) =$
$5 = 2^0 3^0 5^1 = (\,)(\,)((\,)) =$
$6 = 2^1 3^1 = ((\,))((\,)) =$



etc.

by a proper consideration of rightmost paths out of each node, this map can be changed into a one-one onto correspondence; but of course, counting planted plane trees does not a new sequence make.

i have since found out that one Albert A. Mullin had already analyzed integers this way, getting patterns of primes like $10{,}000 = 2^{2^2} 5^{2^2}$ that he called 'mosaics'; and in one place he mentions a graphical correspondence but gives no example. [redacted] articles of his that i can find are in Zeitschr. f. math. Logik und Grundlagen d. Mathk. Bd. 10 p. 159 and p. 199 (1964) and Notre Dame J. of Form. Logic VIII #4 p. 353 (Oct 1967). ~~i think that there are also many abstracts scattered thru the AMS Notices~~ you may want to check with him to see if he knows this sequence[redacted] or has got something similar to riffs. the latest address i've seen is in AMS Notices 26 #2 p. A195 [redacted] (Feb 1979). [redacted] using the number of pts in [redacted] graph of an integer as a measure of its complexity is a study [redacted] related to the measures of 'roundness' discussed in Hardy + Wright.

by way of the Zermelo-Von Neumann construction of integers (as $0 = \phi$, $1 = \{\phi\}$, $2 = \{0, 1\}$, etc), riffs are also a multiplicative analog of Conways surreal numbers + game trees. [redacted] there we have a class of games G recursively defined as ordered pairs of sets of G's, and a special subclass of numbers with nothing on the left $\geq$ anything on the right. here we have a class of forts (forest of oriented rooted trees) F defined as sets of ordered pairs of F's, and a special class of riffs (rooted index-functional forests) in which the riffs above lines into a pt are all distinct[redacted]. another class of graphs in this correspondence, with the same counting sequence, + somewhat easier to draw, are called rotes (rooted odd trees with only exponent symmetries) which are a subset of rooted trees [redacted] formed from $\mathring{\underset{\bullet}{|}}$'s + called gambits. pictures—

(it would be nice if we could add these representations as easily as Conway multiplies his)

integer	factorization	riff	r.i.f.f.	rote → in parentheses
1	[redacted] blank	blank	blank [redacted]	blank
2	p_1^1	p	[redacted] rooted pt	(( ))
3	$p_2^1 = p_{p_1}^1$	p_p		[redacted] ((( ))( ))
4	$p_1^2 = p_1^{p_1}$	p^p		(((( ))))
5	$p_3 = p_{p_2} = p_{p_{p_1}}$	p_{p_p}		((((( ))( ))( ))
6	$p_1 p_2 = p_1 p_{p_1}$	$p p_p$		(( ))((( ))( ))
360	$p_1^3 p_2^2 p_3^1$	$p^{p_p} p_p^p p_{p_p}$		

note that if looked at [redacted] as a set of ordered pairs, where $(A,B) = (A(B))$ then $360 = (1(3))(2(2))(3(1))$, which can be filled [redacted] in recursively to get the parenthetical form of the corresponding rote. also note that rotes are free of any plane embedding as long as the root stays marked; for each pt immediately above the root there is a unique line leading to an odd tree, + this is the exponent. Yours truly, Jon Awbrey

Sent to Martin Gardner in August 1978

The forest primeval: a correspondence between natural numbers and planted plane trees.

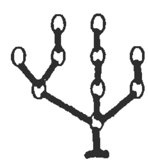
Consider the primes factorization of a number and represent it as the sequence of its exponents separated by parentheses, thus: $N=(n_1)\ (n_2)\ (n_3)\ldots(n_m)$, ignoring for now the all-zero exponents after some point m.

Repeat this process with each exponent in the factorization and with each further exponent in <u>its</u> factorization and so on, until zero exponents are reached along every path of development. A zero exponent, having no representation in primes, may simply be omitted on the next pass. For example:

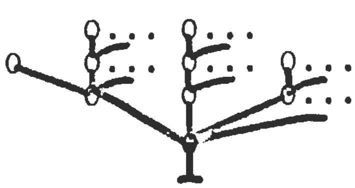
360= (3)(2)(1)=((0)(1))((1))((0))=(()((0)))(((0)))(())
= (()(()))((()))(()).

Thus, parentheses suffice to represent this 'involved factorization' of a number. But they also represent planted plane trees, if we interpret the exterior space as the planter, parenthetical containments as further edges, and contained spaces as nodes, e.g.:

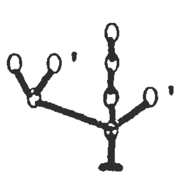
360=



In this way, every number may be assigned a unique minimal tree, but many trees will share the same number when the 'insignificant digits' of a factorization are taken into account. E.g., trees of the following form, where an ellipsis represents an arbitrary number of terminal edges, all have the number 360:



The correspondence becomes one-to-one by the following transformation. To the rightmost 'offspring', when not the only offspring, of any node, assign an extra unit of significance: such that, if terminal, that node represents a unit exponent; if it is not terminal, the unit may be 'inherited', but only by a singleton offspring of that node, with a like condition entailing any further inheritance, until a terminal node is reached; otherwise, the significance is lost. E.g., 360 is now uniquely represented by the following tree, where primed nodes count for unit exponents under the above rule:



Jon Awbrey