Let $a(n,k)=\sum_{i=0}^{n} S(n,i)\cdot\frac{[i]^k}{k!}$, where $S(n,i)$ are Stirling numbers of the second kind and $[i]^k := i(i+1)\ldots(i+k-1), [i]^0=1,$ Pochhammer symbol. Numbers $a(n,k)$, for small values of n and k, are given in the table:

k / n	0	1	2	3	4	5	6	7	8	9
0	1	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1
2	2	3	4	5	6	7	8	9	10	11
3	5	10	16	23	31	40	50	61	73	86
4	15	37	68	109	161	225	302	393	499	621
5	52	151	311	546	871	1302	1856	2551	3406	4441
6	203	674	1530	2906	4957	7859	11810	17031	23767	32288

From the definition of $a(n,k)$ we have:

$$a(1,k)=1,$$

$$a(2,k)=\frac{1}{1!}(k+2),$$

$$a(3,k)=\frac{[1]^k}{k!}+3\frac{[2]^k}{k!}+\frac{[3]^k}{k!}=C(k,k)+3C(k+1,k)+C(k+2,k)=\frac{1}{2!}(k^2+9k+10),$$

$$a(4,k)=\frac{1}{3!}(k^3+24k^2+107k+90),$$

$$a(5,k)=\frac{1}{4!}(k^4+50k^3+575k^2+1750k+1248),$$

$$a(6,k)=\frac{1}{5!}(k^5+90k^4+2135k^3+16050k^2+38244k+24360),\ldots.$$

Exponential generating function (e. g. f.) for numbers $a(n,k)$ is $e^{\frac{e^y-1}{1-x}}$.

Expansion of $e^{\frac{e^y-1}{1-x}}$ with respect to y is

$$1+\frac{1}{1-x}y+\frac{1}{2!}\frac{2-x}{(1-x)^2}y^2+\frac{1}{3!}\frac{5-5x+x^2}{(1-x)^3}y^3+\frac{1}{4!}\frac{15-23x+10x^2-x^3}{(1-x)^4}y^4+$$
$$+\frac{1}{5!}\frac{52-109x+76x^2-19x^3+x^4}{(1-x)^5}y^5+\frac{1}{6!}\frac{203-544x+531x^2-224x^3+36x^4-x^5}{(1-x)^6}y^6+\cdots$$

and expansion of $e^{\frac{e^y-1}{1-x}}$ with respect to x is

$$e^{t-1}(1+(t-1)x+\frac{1}{2!}(t-1)(t+1)x^2+\frac{1}{3!}(t-1)(t^2+4t+1)x^3+\frac{1}{4!}(t-1)(t^3+9t^2+15t-1)x^4+$$
$$+\frac{1}{5!}(t-1)(t^4+16t^3+66t^2+56t-19)x^5+\frac{1}{6!}(t-1)(t^5+25t^4+190t^3+470t^2+185t-151)x^6+\cdots),$$

where $t=e^y$.

Thus o. g. fs. for rows are $1, \frac{1}{1-x}, \frac{2-x}{(1-x)^2}, \frac{5-5x+x^2}{(1-x)^3}, \ldots$ and e. g. fs. for columns are

$$e^{e^x-1}, (e^x-1)\,e^{e^x-1}, \frac{1}{2!}(e^x-1)(e^x+1)\,e^{e^x-1}, \frac{1}{3!}(e^x-1)(e^{2x}+4e^x+1)\,e^{e^x-1},\cdots .$$

In general one can show that o. g. f. for n-th row is $\frac{1}{(1-x)^n}\sum_{j=0}^{n-1}\sum_{i=0}^{n}(-1)^j S(n,n-i)\cdot C(i,j)\,x^j$

and e. g. f. for k-th column is $\frac{1}{k!}\Lambda_k(e^x-1)\,e^{e^x-1}$, where $\Lambda_k(x)=\sum_{i=0}^{k}L'(k,i)\,x^i$ are Lah polynomials

and $L'(k,i)=\frac{k!}{i!}C(k-1,i-1)$, $L'(0,0)=1$, $L'(k,0)=0$, $k>0$, are unsigned Lah numbers.