A055643   Wolfdieter Lang, January 16, 2018.

Explanation of the alternating sexagesimal-decimal number system.

For this Sumerian-Babylonian number system see, e.g., the G. Ifrah reference.

This number system is written here as a positional (place-value) sexagesimal system but with each sexagesimal position replaced by two positions. The positions k >= 0, counted from the right-hand side, have now values 60^l, for l >= 0, if k = 2*l, and 10*60^l, l >= 0, if k = 2*l + 1. See S(k) = A281863(k) for these values for position k. Positions with even k can have the decimal digits 0, 1, …, 9 but for odd k only the digits 0, 1, 2, 3, 4 and 5 are used. The ahistoric use of 0 allows for marking unfilled (empty) positions, and also for the additional representation A055843(0) = 0. Leading zeros are otherwise omitted.

a(n) = A055843(n) = s(K(n)) s(K(n) - 1)...s(0) with the number of digits L(n), the length of a(n), given by L(n) = K(n) + 1. An algorithm A_S for the computation of a(n) operates iteratively on positive numbers m and provides the nonzero digits s(j). For A_S(m) one computes first a quartet [F(m, S), k(m, S), p(m, S), o(m, S)] with F(m, S) = Floor(m, S), the floor function w.r.t. the sequence S = A281863, k(m, S) the position (index) of F(m, S) in S, p(m, S) = floor(m/F(m, S)) and o(m, S) = m – p(m, S)*F(m, S). Then A_S stops if o(m, S) = 0, otherwise it gives the digit s(k(m, S)) = p(m, S), and computes A(o(m, S)). Finally, a(n) is obtained by a(0) :=0, and filling empty positions with 0s.

The two examples form Ifrah (German version) Abb. 157, p. 215 work as follows:

n = 54492:   step 1: 54492 → [F, k, p, o] = [36000, 5, 1, 18492], computes s(5) = 1; step 2: 18492 → [3600, 4, 5, 492], s(4) = 5; step 3: 492 → [60, 2, 8, 12], s(2) = 8; step 4: 12 → [10, 1, 1, 2], s(1) = 1; step 5: 2 → [1, 0, 2, 0], s(0) = 2, and then A_S stops because o became 0. The position k=3 did not appear (in the Ifrah example the entry for 600 was not used), it has to be filled with 0 in order to obtain a(n) = 150812.

n = 199539:   step1: [36000, 5, 5, 19539], s(5) = 5, step 2: [3600, 4, 5, 1539], s(4) = 5, step 3:   [600, 3, 2, 339], s(3) = 2; step 4:   [60, 2, 5, 39], s(2) =5; step 5:   [10, 1, 3, 9], s(1) = 3; step 6:   [1, 0, 9, 0], s(0) = 9 ; and A_S stops because o =0. All positions from k = 5 on have been used, no 0's were in the game: a(n) = 552539. (In the Ifrah cuneiform figure the line has been broken to show the last two 3 and 9 symbols)

The number of digits of a(n), its length L(n), is given by L(n) = A282622.

This number system has been explained already in a now recycled entry A282621 from Feb 20 2017. A282621 has been recycled because the numbers duplicated A055643(n), for n >= 1.

References:

Georges Ifrah, Histoire Universelle des Ciffres, Paris, 1981.

Georges Ifrah, From one to zero, A universal historf numbers, Viking Penguin Inc., 1985.

Georges Ifrah, Universalgeschichte der Zahlen, Campus Verlag, Frankfurt, New York, 2. Auflage, 1987, pp. 210-221.