WayBack Machine	http://2000clicks.com/MathHelp/BasicSequenceA049982.htm Lcanture 22 Nov 2006	Co OCT NOV DEC 2007 2008 2009		Accur this capture Accur this capture
		2	A049982	

Math Help -> Basic Principles -> Sequences -> A049982

A049982 is the number of arithmetic progressions of 2 or more positive integers, strictly increasing with sum n.

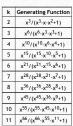
When I stumbled on this sequence (and it's brothers and sisters, with various slightly different qualifications), I noticed a complete lack of any formulas or generating functions that help understand the sequence. So I did some amateur investigation on my own

I started by considering the number of arithmetic progressions of 2 positive integers, strictly increasing with sum n. By convention, I like to start with n=0, so this sequence is 0, 0, 0, 1, 1, 2, 2, 3, 3, 4, 4, ... which has generating function $x^3/(x^3 \cdot x^2 \cdot x \cdot 1)$ which I will rewrite as $x^3/(x^3 \cdot x \cdot x^2 + 1)$ for reasons that will become clear later.

The sum of an arithmetic progression of 3 positive integers is always three times its middle term, hence a multiple of 3. This sequence is 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 3, 0, 0, 4, ... which has generating function $x^6/(x^6-2x^3+1)$, which I will write as $x^6/(x^6-x^3+1)$ for reasons that will become clear later

As you can imagine, I kept going. As the going got tougher, I started inventing little tools to help, such as PuzzleGeneratingFunction.xis, an Excel spreadsheet that guesses the generating function for a given series. (..... maybe III write a page about that spreadsheet some day.) After a while, I had a little table that shows the

generating function for the sequence of the number of arithmetic progressions of k positive integers, strictly increasing with sum n



Now, maybe you can see why I wrote the terms for k=2 and k=3 in such a funny way. In general, the generating function for the sequence of the number of arithmetic progressions of k positive integers, strictly increasing with sum n is:

 $x^{t(k)}/(x^{t(k)}\cdot x^{t(k\cdot 1)}\cdot x^{k} \cdot 1),$ where t(k) is the k'th triangular number

Summary

A049982 has generating function $x^3/(x^3 \cdot x \cdot x^2 \cdot 1) + x^6/(x^6 \cdot x^3 \cdot x^3 + 1) + x^{10}/(x^{10} \cdot x^6 \cdot x^4 \cdot 1) + \dots$ which is the sum k=2,3,... of $x^{k(k)}/(x^{k(k)} \cdot x^{k(k-1)} \cdot x^{k-1})$, where t(k) is the kth triangular number Term k of this generating function generates the number of arithmetic progressions of k positive integers, strictly increasing with sum n.

Internet References

A049982 -- The Online Encyclopedia of Integer Sequences.

Related pages in this website

See also <u>Recurrence Relation</u>

The webmaster and author of the <u>Math Help</u> site is Graeme McRae. [home] [email] [search] [Links to Math Sites] [Whiteboard]

