Cinquante signes

The Ghost Iteration

novembre 09, 2019

El Greco, *The Vision of Saint John* (1610-1614)

Take an integer like `2019`.

We will call "ghost" the concatenation of the absolute differences of its digits.

The *ghost* of `2019` is thus `218`.

(We don't keep the possible leading zeros of a *ghost*: for example, the *ghost* of `1186` is `72` (not `072`).

The A rule:
If a *ghost* is even, add it to the starting integer. Else subtract it. And iterate from there.

```
  2019
+  218   <-- first ghost
------
  2237
+   14   <-- second ghost
------
  2251
+   34   <-- third ghost
------
  2285
-   63   <-- fourth ghost
------
  2222
=    0   <-- last and zero ghost
```

It seems that every integer will have a *zero ghost* at the end of the process. And this seems to be true even with the *opposite* **B rule**.

The B rule:
If a *ghost* is odd, add it to the starting integer. Else subtract it. And iterate from there.

```
  2019
-  218   <-- first ghost
------
  1801
+  781   <-- second ghost
------
  2582
-  336   <-- third ghost
------
```

```
  2246
-   22   <-- fourth ghost
------
  2224
-    2   <-- fifth ghost
------
  2222
=    0   <-- last and zero ghost
```

Now, how could we submit this *Ghost Iteration* to the OEIS? We must note that a single-digit integer has no ghost: 8, for instance, lives by itself.
Let's see what the **A rule** produces with the successive integers > 9:
"If a *ghost* is even, add it to the starting integer. Else subtract it. And iterate from there".

```
10 - 1 = 9 --> END (single digit)
11 --> zero ghost = END
12 - 1 = 11 (END, see above)
13 + 2 = 15
         15 + 4 = 19
                  19 + 8 = 27
                           27 - 5 = 22 (zero ghost, END)
14 - 3 = 11 (zero ghost, END)
15 (see above)
16 - 5 = 11 (zero ghost, END)
17 + 6 = 23
         23 - 1 = 22 (zero ghost, END)
18 - 7 = 11 (zero ghost, END)
19 (see above)
20 + 2 = 22 (zero ghost, END)
21 - 1 = 20 (see above)
22 (zero ghost, END)
23 - 1 = 22 (zero ghost, END)
24 + 2 = 26
```

```
          26 + 4 = 30
                   30 - 3 = 27
                            27 - 5 = 22 (zero ghost, END)
25 - 3 = 22 (zero ghost, END)
26 (see above)
27 (see above)
28 + 6 = 34
          34 - 1 = 33 (zero ghost, END)
29 - 7 == 22 (zero ghost, END)
30 (see above)
31 + 2 = 33 (zero ghost, END)
etc.
```

We could assign to every n > 9 the number of iterations before reaching an END:

```
10 ~ 1
11 ~ 0
12 ~ 1
13 ~ 4
14 ~ 1
15 ~ 3
16 ~ 1
17 ~ 2
18 ~ 1
19 ~ 2
20 ~ 1
21 ~ 1
22 ~ 0
23 ~ 1
24 ~ 4
25 ~ 1
26 ~ 3
27 ~ 1
```

```
28 ~ 2
29 ~ 1
30 ~ 2
31 ~ 1
etc.
```

This sequence could be defined like that:
> Number of steps such that n+9 reaches an END in the *Ghost Iteration* (see the Comments section).
S = 1,0,1,4,1,3,1,2,1,2,1,1,0,1,4,1,3,1,2,1,2,1,...
Yes, there is a pattern there:
S = 1,0,1,4,1,3,1,2,1,2,1,1,0,1,4,1,3,1,2,1,2,1...

A similar sequence could be submitted for the **B rule**.

Note that if an integer enters a loop, it will have no ghost (see below); the S sequence should thus have a minus one (-1) value for such *n*s.
Last word: some integers have many ancestors, no matter the rule — but this is another (fun) story.

Best,
É.

11th nov 2019 update.

Maximilian Hasler was kind enough to put the **A** rule here, and the **B** rule there (OEIS).
Harvey P. Dale was the first to spot an integer entering a loop (thus having no 'zero ghost'): `11090`.
This integer can produce an infinite famiy of such 'loopers' by preceding `11090` by any quantity of 1-repunits: `111090, 1111090, 11111090`, etc.
More *loopers* were found a few hours later by Maximilian and Lars Blomberg (see below)

11th nov 2019 second update

Maximilian Hasler has put a lot of stuff on the OEIS (follow the links, read and enjoy!-)

Here is the « Irregular table whose rows list the nontrivial cycles of the ghost iteration A329200 [**rule A**] starting with the smallest member »:
10891, 12709, 11130, 11107, 11090, 43600, 44960, 45496, 44343, 44232, 44021, 74780, 78098, 76207, 75800, 78180, 79958, 77915, 78199, 79979, 82001, 110891, 112709, 111130, 111107, 111090, 180164, 258316, 224791, 227119, 232727, 221172, 220107, 217990, 201781,...

Here is the « Irregular table whose rows list the nontrivial cycles of the ghost iteration A329201 [**rule B**], starting with the smallest member »:
8290, 8969, 9102, 17998, 24199, 21819, 20041, 22084, 21800, 20020, 21901, 23792, 25219, 54503, 55656, 55767, 55978, 56399, 55039, 87290, 88869, 88892, 88909, 89108, 108070, 126947, 141300, 221901, 223792, 225219, 554503, 555656, 555767, 555978, 556399, 555039,...

Here is a list of 107 *loopers* (**rule A**) sent to me by **Lars Blomberg** , confirming Harvey's and Maximilian's computations:

10891, 11090, 11107, 11130, 12709, 43600, 44021, 44232, 44343, 44960, 45496, 74780, 75800, 76207, 77915, 78098, 78180, 78199, 79958, 79979, 82001, 110891, 111090, 111107, 111130, 112709, 180164, 201781, 217990, 220107, 221172, 224791, 227119, 232727, 258316, 443600, 444021, 444232, 444343, 444960, 445496, 774780, 776207, 778098, 858699, 873052, 891929, 1110891, 1111090, 1111107, 1111130, 1112709, 3270071, 3301514, 3427147, 4381182, 4412625, 4443600, 4444021, 4444232, 4444343, 4444960, 4445496, 4538258, 5492293, 5523736, 5649369, 7774780, 7776207, 7778098, 8858699, 8873052, 8891929, 11110891, 11111090, 11111107, 11111130, 11112709, 33270071, 33301514, 33427147, 44381182, 44412625, 44443600, 44444021, 44444232, 44444343, 44444960, 44445496, 44538258, 55492293, 55523736, 55649369, 77774780, 77776207, 77778098, 85869922, 87305285, 88858699, 88873052, 88891929, 89192992, 111110891, 111111090, 111111107, 111111130, 111112709,...

I couln't find any integer that would loop under both rules **A** and **B**, though. Looks impossible to me...
Again, many thanks to all!
Best,
É.

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Posts les plus consultés de ce blog

A square for three (chess)

juin 22, 2024

(English translation after the French text) Voici cinq problèmes d'échecs disjoints : a) combien faut-il de coups au minimum pour que trois pions soient capturés sur la même case ? b) trois tours c) trois c ...

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Le tripalin se présente

avril 11, 2024

Un tripalin est constitué de trois images. Chaque image illustre un substantif. Accolés, ces trois substantifs forment une chaîne palindromique. Laquelle nous vous invitons à trouver. Exem ...

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Some strings au cinéma Galeries

juillet 19, 2024

Lettre ouverte au cinéma Galeries Bonsoir à tous, Je viens de voir pour la seconde fois chez vous le beau film de Léos Carax (la première fois c'était le 26 juin en présence du réalisateur, au BRIFF). Apparut à l'écran aujourd'hui, avant la projection propre ...

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