

Wolfdieter Lang, Jul 29 2013
 a(n,k) tabf head (staircase) for A036038

M1 (or M_1) multinomial numbers for partitions of n in Abramowitz-Stegun (A-St) order.

$$M1([a_1, \dots, a_n]) = n! / \text{product}(j!^{a_j}, j=1..n).$$

The row number is n and m is the number of parts of a partition of n.

k numbers the partitions in the A-ST order.

n\k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	...
1	1																														
2	1	2																													
3	1	3	6																												
4	1	4	6	12	24																										
5	1	5	10	20	30	60	120																								
6	1	6	15	20	30	60	90	120	180	360	720																				
7	1	7	21	35	42	105	140	210	210	420	630	840	1260	2520	5040																
8	1	8	28	56	70	56	168	280	420	560	336	840	1120	1680	2520	1680	3360	5040	6720	10080	20160	40320									
9	1	9	36	84	126	72	252	504	630	756	1260	1680	504	1512	2520	3780	5040	7560	3024	7560	10080	15120	22680	15120	30240	45360	60480	90720	181440	362880	
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n\k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	...

The 42 members of row n=10 are:

1, 10, 45, 120, 210, 252, 90, 360, 840, 1260, 1260, 2520, 3150, 4200, 720, 2520, 5040, 6300, 7560, 12600, 16800, 18900, 25200, 5040, 15120, 25200, 37800, 50400, 75600, 113400, 30240, 75600, 100800, 151200, 226800, 151200, 302400, 453600, 604800, 907200, 1814400, 3628800.

The sequence of row lengths is A000041: [1, 2, 3, 5, 7, 11, 15, 22, 30, 42,...] (partition numbers), n >= 1.

The sequence of row sums is A005651: [1, 3, 10, 47, 246, 1602, 11481, 95503, 871030, 8879558,...], n >= 1.

The sequence of alternating row sums is: [1, -1, 4, 15, 76, 470, 3207, -25697, -234404, -2341806,...], n >= 1.

One could add the row for n=0 with a 1, if the part 0 is considered for n=0, and only for this n.

For the ordering of this tabf array a(n,k) see Abramowitz-Stegun ref. pp. 831-2.

E.g. a(5,4) refers to the fourth partition of n=5 in this ordering, namely (1^2,3^1) = [1,1,3], whence a(5,4) = 20, because 5!/(1!^2*3!^1) = 120/6 = 20. Or a(6,5) = 30 for the partition (1^2,4^1)=[1,1,4] of n=6 from the same computation.

a(7,10) = 420 from the 10th partition of n=7 which is (1^2,2^1,3^1)=[1,1,2,3] with 7!/(1!^2*2!^1*3!^1) = 420.

e.o.f.#####